OXFORD
INTERNATIONAL
AQA EXAMINATIONS

INTERNATIONAL AS MATHEMATICS

(9660)

Mark scheme

Pure Mathematics Unit 1

Specimen

Copyright © 2017 Oxford International AQA Examinations and its licensors. All rights reserved.

Principal Examiners have prepared these mark schemes for specimen papers. These mark schemes have not, therefore, been through the normal process of standardising that would take place for live papers.

Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Copyright © 2017 Oxford International AQA Examinations and its licensors. All rights reserved.

No method shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Copyright © 2017 Oxford International AQA Examinations and its licensors. All rights reserved.

Q	Answer	Marks	Comments
1(a)(i)	–3	B1ft	ft on $-p$
1(a)(ii)	$\frac{1}{2}$	B1	oe
1(b)	$2^{\frac{1}{2}} \times 2^x = 2^{-3} \Rightarrow 2^{\frac{1}{2}+x} = 2^{-3}$	M1	Using a law of indices or logs correctly to combine at least two of the powers of 2 PI
	$\Rightarrow x = -3\frac{1}{2}$	A1ft	ft on $x = q - r$ if M1 awarded
	Total	**4**	

Q	Answer	Marks	Comments
2(a)	$y = \frac{13}{3} - \frac{7}{3}x$	M1	attempt at $y = a + bx$ or $\frac{\Delta y}{\Delta x}$ with 2 correct points
	(gradient =) $-\frac{7}{3}$	A1	condone slip in rearranging if gradient is correct
2(b)(i)	$y - 3 =$ 'their grad'$(x - -1)$	M1	or $7x + 3y = k$ and attempt at k using $x = -1$ and $y = 3$ or $y =$ (their m)$x + c$ and attempt at c using $x = -1$ and $y = 3$
	$y - 3 = -\frac{7}{3}(x + 1)$ or $7x + 3y = 2$ or $y = -\frac{7}{3}x + \frac{2}{3}$	A1cso	correct equation in any form and replacing – – with + sign
2(b)(ii)	(4, –5)	B1,B1	$x = 4$, $y = -5$ withhold if clearly from incorrect working
2(c)	$7x + 3y = 13$ and $3x + 2y = 12$ $\Rightarrow$ equation in x or y only	M1	must use correct pair of equations and attempt to eliminate y (or x)
	$x = -2$	A1	
	$y = 9$	A1	
	Total	**8**	

Copyright © 2017 Oxford International AQA Examinations and its licensors. All rights reserved.

Q	Answer	Marks	Comments
3(a)	$(2 + x^2)^3$ $= [(2)^3] + 3(2)^2(x^2) + 3(2)(x^2)^2\ [+ (x^2)^3]$	M1	For either (1),3,3,(1) oe unsimplified or $\binom{3}{1} 2^2x^2 + \binom{3}{2} 2(x^2)^2$ oe PI
	$p = 3(2)^2 = 12$	A1	AG Be convinced Accept left as $12x^2$
	$q = 6$	B1	Accept left as $6x^4$
3(b)(i)	$\int \frac{(2+x^2)^3}{x^4} \mathrm{d}x =$ $\int x^{-4}(8 + 12x^2 + qx^4 + x^6)\,\mathrm{d}x$ or $\int \left(\frac{8}{x^4} + \frac{12}{x^2} + q + x^2\right)\mathrm{d}x$	M1	Uses (a) and either an indication that $\frac{1}{x^n} = x^{-n}$ in a product PI or cancelling to get at least 3 correct ft terms
	$\int (8x^{-4} + 12x^{-2} + q + x^2)\,\mathrm{d}x$	A1ft	ft on their non-zero q PI by next line in solution
	$= \frac{8x^{-3}}{-3} + \frac{12x^{-1}}{-1} + qx + \frac{x^3}{3}\ \{+c\}$	M1	Correct integration of either $8x^{-4}$ or $12x^{-2}$, accept unsimplified
		A1	Correct integration of both $8x^{-4}$ and $12x^{-2}$, accept unsimplified coefficients
	$= \ldots\ldots\ldots\ldots\ldots\ldots\ldots + 6x + \frac{x^3}{3} + c$ $(= -\frac{8}{3}x^{-3} - 12x^{-1} + 6x + \frac{x^3}{3} + c)$	B1ft	For "6" $x + \frac{x^3}{3} + c$ simplified The only ft is "6" replaced by their value for q where q is a non-zero integer

Copyright © 2017 Oxford International AQA Examinations and its licensors. All rights reserved.

Q	Answer	Marks	Comments
3(b)(ii)	$\int_1^2 \frac{(2+x^2)^3}{x^4}\,dx =$ $\left\{-\frac{8}{3}(2)^{-3} - 12(2^{-1}) + 6(2) + \frac{2^3}{3}\right\} -$ $\left\{-\frac{8}{3}(1)^{-3} - 12(1)^{-1} + 6(1) + \frac{1^3}{3}\right\}$ $= \left(-\frac{1}{3} - 6 + 12 + \frac{8}{3}\right) - \left(-\frac{8}{3} - 12 + 6 + \frac{1}{3}\right)$	M1	Dealing correctly with limits; F(2)−F(1) (must have attempted integration to get F ie their F is not just the integrand)
	$= 16\frac{2}{3}$	A1	oe **exact** answer eg 50/3 NMS scores 0
	Total	**10**	

Q	Answer	Marks	Comments
4(a)(i)	$ar^2 = 36$; $ar^5 = 972$;	M1	For $ar^2 = 36$ or $ar^5 = 972$ or for seeing $36r^3 = 972$
	$r^3 = \frac{972}{36}$ (= 27) $\Rightarrow r = 3$	A1	CSO AG Full valid completion
4(a)(ii)	$a \times 3^2 = 36$	M1	oe PI
	$a = 4$	A1	Correct answer without working scores the two marks
4(b)	$\sum_{n=1}^{20} u_n = S_{20} = \frac{a(1-r^{20})}{1-r}$	M1	oe
	$= \frac{4(1-3^{20})}{-2} = -2(1-3^{20}) = 2(3^{20}-1)$	A1	CSO AG Be convinced
	Total	**6**	

Copyright © 2017 Oxford International AQA Examinations and its licensors. All rights reserved.

Q	Answer	Marks	Comments
5	$h = 0.5$	B1	PI by x-values 0, 0.5, 1, 1.5 provided no contradiction
	$\mathrm{f}(x) = \sqrt{27x^3 + 4}$ $\mathrm{I} \approx \frac{h}{2}\{\mathrm{f}(0) + \mathrm{f}(1.5) + 2[\mathrm{f}(0.5) + \mathrm{f}(1)]\}$	M1	OE summing of areas of the 'trapezia'.
	$\frac{h}{2}$ with $\{\} = 2 + \sqrt{95.125} + 2(\sqrt{7.375} + \sqrt{31})$ $= \frac{h}{2}\{2 + 9.753... + 2[2.715... + 5.567...]\}$	A1	OE Accept 2dp rounded or truncated or better as evidence for surds. Can be implied by later <u>correct</u> work provided >1 term or a single term for {…} which rounds to 28.32
	$(\mathrm{I} \approx 0.25 \times 28.320...)$ $(= 58.25(119...))$ $\mathrm{I} = 7.08$ (to 3 sf)	A1	CAO Must be 7.08
	Total	**4**	

6(a)	$\mathrm{p}(3) = 3^3 - 2 \times 3^2 + 3$ $(= 27 - 18 + 3)$	M1	p(3) attempted; not long division
	$= 12$	A1	
6(b)	$\mathrm{p}(-1) = (-1)^3 - 2(-1)^2 + 3$	M1	p(–1) attempted; not long division
	$\mathrm{p}(-1) = -1 - 2 + 3 = 0 \Rightarrow x + 1$ is a factor	A1cso	correctly shown = 0 plus statement
6(c)(i)	Quadratic factor $(x^2 - 3x + 3)$	M1	$b = -3$ or $c = 3$ by inspection or full long division attempt or comparing coefficients
	$(\mathrm{p}(x) =)$ $(x + 1)(x^2 - 3x + 3)$	A1	must see correct product
6(c)(ii)	Discriminant of quadratic $b^2 - 4ac = (-3)^2 - 4 \times 3$	M1	their discriminant considered possibly within quadratic equation formula
	$b^2 - 4ac < 0 \Rightarrow$ no real roots from quadratic $\Rightarrow$ only one real root	A1cso	
	Total	**8**	

Copyright © 2017 Oxford International AQA Examinations and its licensors. All rights reserved.

Q	Answer	Marks	Comments
7(a)	$\{S_{25} =\}\ \frac{25}{2}[2a + (25-1)d]$	M1	$\frac{25}{2}[2a + (25-1)d]$ oe
	$\frac{25}{2}[2a + 24d] = 3500$ $25(2a + 24d) = 7000$ or $\left[\frac{50a + 600d}{2} = 3500\right]$	m1	Forming equation and attempt to remove fraction or to expand brackets or better
	$50a + 600d = 7000$ (or better) so $a + 12d = 140$	A1	CSO AG Be convinced
7(b)	5th term = $a + 4d$	M1	$a + (5-1)d$ used correctly
	$a + 12d = 140,\ a + 4d = 100$ $\Rightarrow 8d = 40$	M1	Solving $a + 12d = 140$ simultaneously with either $a + 4d = 100$ or $a + 5d = 100$ as far as eliminating either a or d
	$\Rightarrow d = 5$	A1	
	$\Rightarrow a = 80$	A1	
7(c)	$33\left(3500 - \sum_{n=1}^{k} u_n\right) = 67\sum_{n=1}^{k} u_n$	m1	Recognition that $\sum_{n=1}^{25} u_n = 3500$
	$33 \times 3500 = 67\sum_{n=1}^{k} u_n + 33\sum_{n=1}^{k} u_n$	M1	Correct rearrangement PI
	$100 \times \sum_{n=1}^{k} u_n = 33 \times 3500 \Rightarrow \sum_{n=1}^{k} u_n = 1155$	A1	
	Total	10	

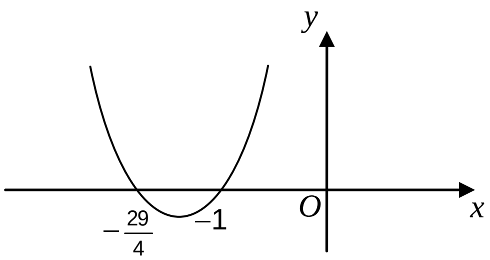
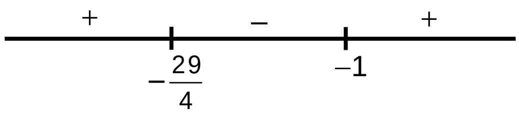
Copyright © 2017 Oxford International AQA Examinations and its licensors. All rights reserved.

Q	Answer	Marks	Comments
8(a)	$\int_{-1}^{1}(x^3 - 2x^2 + 3)\,dx$		
	$= \left[\frac{x^4}{4} - \frac{2x^3}{3} + 3x\right]_{-1}^{1}$	M1	One term correct
		A1	Another term correct
		A1	All correct (condone + c)
	$= \left(\frac{1}{4} - \frac{2}{3} + 3\right) - \left(\frac{1}{4} + \frac{2}{3} - 3\right)$	B1ft	'their' F(1) – F(–1) with $(-1)^3$ etc evaluated correctly but must have earned M1
	$= 4\frac{2}{3}$	A1cso	$\frac{14}{3}$, $\frac{56}{12}$ etc but combined as single fraction
8(b)	Area of $\Delta\left(= \frac{1}{2} \times 2 \times 2\right)$ $= 2$	B1	PI
	Shaded region has area $4\frac{2}{3} - 2$	M1	± their (a) ± their Δ area
	$= 2\frac{2}{3}$	A1cso	$\frac{8}{3}$, $\frac{32}{12}$ etc but combined as single fraction
	Total	8	

Copyright © 2017 Oxford International AQA Examinations and its licensors. All rights reserved.

Q	Answer	Marks	Comments
9(a)(i)	(When x = 2) $\frac{dy}{dx}$ = 12 – 1 – 11 = 0	B1	AG Must see intermediate evaluations
9(a)(ii)	$\frac{4}{x^2} = 4x^{-2}$ {so $\frac{dy}{dx} = 3x^2 - 4x^{-2} - 11$}	B1	$\frac{4}{x^2} = 4x^{-2}$, seen in **(a)(ii)** or earlier. PI by ± $8x^{-3}$ term in answer
	$\frac{d^2y}{dx^2} = 6x + 8x^{-3}$	M1	Correct powers of x correctly obtained from differentiating the first two terms
		A1	$6x + 8x^{-3}$ ACF
	When x = 2, $\frac{d^2y}{dx^2}$ = 12 + 8/8 = 13	A1	
9(a)(iii)	Since $\frac{d^2y}{dx^2} > 0$, P is a minimum point	E1ft	ft their value of $y''(2)$ in **(a)(ii)** but must see reference to sign of $y''(2)$ either explicitly or as inequality, as well as the correct ft conclusion
9(b)	$\int\left(3x^2 - \frac{4}{x^2} - 11\right) = x^3 + 4x^{-1} - 11x\ (+\ c)$	M1	Attempt to integrate $\frac{dy}{dx}$ with at least two of the three terms integrated correctly
	(y =) $x^3 + 4x^{-1} - 11x\ (+\ c)$	A1	For $x^3 + 4x^{-1} - 11x$ oe even unsimplified
	When x = 2, y = 1 ⇒ 1 = 8 + 2 – 22 + c	M1	Substituting. x = 2, y = 1 into y = F(x) + 'c' in attempt to find constant of integration, where F(x) follows attempted integration of expression for $\frac{dy}{dx}$
	$y = x^3 + 4x^{-1} - 11x + 13$	A1	ACF
	Total	**10**	

Copyright © 2017 Oxford International AQA Examinations and its licensors. All rights reserved.

Q	Answer	Marks	Comments
10(a)(i)	$(-)\ (x + 5)^2$	M1	$q = 5$; condone $(-x - 5)^2$
	$29 - (x + 5)^2$	A1	$p = 29$ and $q = 5$
10(a)(ii)	$x = -5$ is line of symmetry	B1ft	ft $x = -$ their q or correct
10(b)(i)	$4 - 10x - x^2 = k\,(4x - 13)$ $\Rightarrow x^2 + 4kx + 10x - 13k - 4 = 0$ $\Rightarrow x^2 + 2(2k + 5)x - (13k + 4) = 0$	B1	Must see both these lines OE AG all correct working and = 0
10(b)(ii)	2 distinct roots $\Rightarrow b^2 - 4ac > 0$	B1	stated or used (must be > 0)
	Discriminant = $4(2k + 5)^2 + 4(13k + 4)$ $4(2k^2 + 20k + 25 + 13k + 4) > 0$	M1	condone one slip (may be within formula) or $16k^2 + 132k + 116 > 0$
	$\Rightarrow 4k^2 + 33k + 29 > 0$	A1	AG > 0 must appear before final line
10(b)(iii)	$(4k + 29)(k + 1)$	M1	correct factors or correct unsimplified quadratic equation formula $\dfrac{-33 \pm \sqrt{33^2 - 4 \times 4 \times 29}}{8}$
	$k = -\dfrac{29}{4}$, $k = -1$	A1	condone $k = -\dfrac{58}{8}$, -7.25 etc but not left with square roots etc as above
		M1	sketch or sign diagram including values 
	$k < -\dfrac{29}{4}$, $k > -1$	A1	condone use of **OR** but not **AND**
	Take their final line as their answer		
	Total	**11**	

Copyright © 2017 Oxford International AQA Examinations and its licensors. All rights reserved.

GET HELP AND SUPPORT

Visit our website for information, guidance, support and resources at **oxfordaqaexams.org.uk**

ou can contact the English subject team directly at:

E: **english@oxfordaqaexams.org.uk**

OXFORD INTERNATIONAL AQA EXAMINATIONS
LINACRE HOUSE, JORDAN HILL, OXFORD, OX2 8TA
UNITED KINGDOM
enquiries@oxfordaqaexams.org.uk
oxfordaqaexams.org.uk

Copyright © 2017 Oxford International AQA Examinations and its licensors. All rights reserved.