OXFORD
INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number						Candidate number				

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________

INTERNATIONAL A-LEVEL
MATHEMATICS

(9660/MA03) Unit P2 − Pure Mathematics

Specimen 2018 Morning Time allowed: 2 hours 30 minutes

Materials
- For this paper you must have the booklet of formulae and statistical tables.
- You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use a supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box or around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 120.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions in the spaces provided.

1 A curve has equation $y = x^3 \ln x$

1 (a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

[2 marks]

Answer ____________________________

1 (b) (i) Find an equation of the tangent to the curve $y = x^3 \ln x$ at the point on the curve where $x = \mathrm{e}$

[3 marks]

Answer ____________________________

1 (b) (ii) This tangent intersects the x-axis at the point A.

Find the exact value of the x-coordinate of the point A.

[2 marks]

Answer ______________________________

2 (a) (i) Express $\dfrac{5x-6}{x(x-3)}$ in the form $\dfrac{A}{x}+\dfrac{B}{x-3}$

[2 marks]

Answer

2 (a) (ii) Find $\displaystyle\int \frac{5x-6}{x(x-3)}\,\mathrm{d}x$

[2 marks]

Answer

2 (b) (i) Given that

$$4x^3 + 5x - 2 = (2x + 1)(2x^2 + px + q) + r$$

find the values for the constants p, q and r.

[4 marks]

$p =$ ______ $q =$ ______ $r =$ ______

2 (b) (ii) Find $\int \frac{4x^3 + 5x - 2}{2x + 1}\, dx$

[3 marks]

3 (a) Express $\sin x - 3\cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving your value of α to the nearest 0.1°.

[3 marks]

3 (b) Hence find the values of x in the interval $0° < x < 360°$ for which

$$\sin x - 3\cos x + 2 = 0$$

giving your values of x to the nearest degree.

[4 marks]

Answer

Turn over for the next question

4 The functions f and g are defined with their respective domains by

$$f(x) = \sqrt{2x - 5}, \quad \text{for } x \geqslant 2.5.$$

$$g(x) = \frac{10}{x}, \quad \text{for all real values of } x,\ x \neq 0$$

4 (a) State the range of f.

[2 marks]

4 (b) (i) Find $fg(x)$

[1 mark]

4 (b) (ii) Solve the equation $fg(x) = 5$

[2 marks]

4 (c) The inverse of f is f^{-1}

4 (c) (i) Find $\mathrm{f}^{-1}(x)$

[3 marks]

4 (c) (ii) Solve the equation $\mathrm{f}^{-1}(x) = 7$

[2 marks]

5 (a) (i) Find the binominal expansion of $(1 - x)^{\frac{1}{3}}$ up to and including the term in x^2.

[2 marks]

5 (a) (ii) Hence, or otherwise, show that

$$(125 - 27x)^{\frac{1}{3}} \approx 5 + \frac{m}{25}x + \frac{n}{3125}x^2$$

for small values of x, stating the values of the integers m and n.

[3 marks]

5 (b) Use your result from part (a)(ii) to find an approximate value of $\sqrt[3]{119}$, giving your answer to five decimal places.

[2 marks]

Answer

6 (a) (i) Solve the equation $\operatorname{cosec} \theta = -4$ for $0° < \theta < 360°$, giving your answers to the nearest 0.1°

[2 marks]

Answer

6 (a) (ii) Solve the equation

$$2\cot^2(2x + 30°) = 2 - 7\operatorname{cosec}(2x + 30°)$$

for $0° < x < 180°$, giving your answers to the nearest 0.1°

[6 marks]

Answer

6 (b) Describe a sequence of **two** geometrical transformations that maps the graph of $y = \text{cosec}\, x$ onto the graph of $y = \text{cosec}\,(2x + 30°)$.

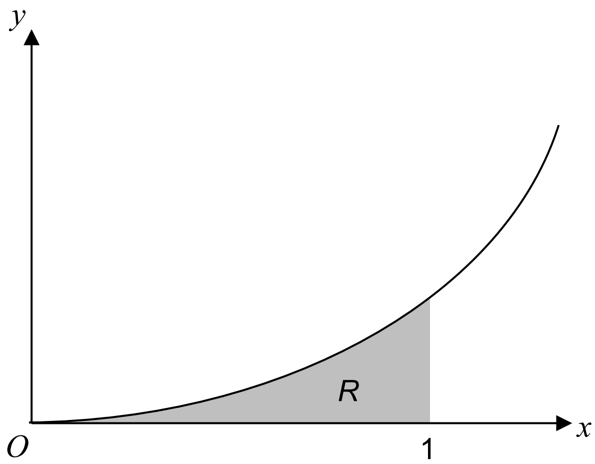
[4 marks]

7 (a) By using integration by parts, find $\int x\,\mathrm{e}^{6x}\,\mathrm{d}x$

[4 marks]

Answer

7 (b) The diagram shows part of the curve with equation $y = \sqrt{x}\,e^{3x}$



The shaded region R is bounded by the curve $y = \sqrt{x}e^{3x}$, the line $x = 1$ and the x-axis from $x = 0$ to $x = 1$.

Find the volume of the solid generated when the region R is rotated through 360° about the x-axis, giving your answer in the form $(pe^6 + q)\pi$, where p and q are rational numbers.

[3 marks]

Answer

8 A curve is defined by the parametric equations $x = 3\cos 2\theta,\ y = 2\cos\theta$

8 (a) (i) Show that $\dfrac{dy}{dx} = \dfrac{1}{k\cos\theta}$, where k is an integer.

[4 marks]

8 (a) (ii) Find an equation of the normal to the curve at the point where $\theta = \dfrac{\pi}{3}$

[4 marks]

8 (b) Find the exact value of $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x \, \mathrm{d}x$

[5 marks]

9 A curve is defined by the equation $9x^2 - 6xy + 4y^2 = 3$

Find the coordinates of the two stationary points of this curve.

[8 marks]

Turn over for the next question

10 The value, £V, of an initial investment, £P, at the end of n years is given by the formula

$$V = P\left(1 + \frac{r}{100}\right)^n$$

where r % per year is the fixed interest rate.

Mr Green invests £1000 in Barcelona Bank at a fixed interest rate of 3% per year.

10 (a) (i) Find the value of Mr Green's investment at the end of 5 years.

Give your value to the nearest £10.

[1 mark]

10 (a) (ii) If Mr Green keeps his money invested for N years, where N is a whole number, find the value for N for which the value of his investment will first exceed £2000.

[3 marks]

Answer

10 (b) Mr White invests £1500 in Bilbao Bank at a fixed interest rate of 1.50% per year.

Mr Green and Mr White invested their money at the same time.

The value of Mr Green’s investment first exceeded the value of Mr White’s investment after T complete years.

Find the value of T.

[4 marks]

Answer ______________________

11 The points A and B have coordinates (5, 1, –2) and (4, –1, 3) respectively.

The line l has equation $\mathbf{r} = \begin{bmatrix} -8 \\ 5 \\ -6 \end{bmatrix} + \mu \begin{bmatrix} 5 \\ 0 \\ -2 \end{bmatrix}$

11 (a) Find a vector equation of the line that passes through A and B.

[3 marks]

Answer ______________________

11 (b) (i) Show that the line that passes through A and B intersects the line l, and find the coordinates of the point of intersection, P.

[4 marks]

11 (b) (ii) The point C lies on l, such that triangle PBC has a right angle at B.

Find the coordinates of C.

[5 marks]

12 (a) A water tank has a height of 2 metres. The depth of the water in the tank is h metres at time t minutes after water begins to enter the tank. The rate at which the depth of water in the tank increases is proportional to the difference between the height of the tank and depth of the water.

Write down a differential equation in the variables h and t and a positive constant k.

(You are not required to solve your differential equation.)

[3 marks]

Answer

12 (b) (i) Another water tank is filling in such a way that *t* minutes after the water is turned on, the depth of water, x metres, changes according to the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{15x\sqrt{2x-1}}$$

The depth of water is 1 metre when the water is first turned on.

Solve this differential equation to find t as a function of x.

[8 marks]

Answer ____________________

12 (b) (ii) Calculate the time taken for the depth of water to reach 2 metres, giving your answer to the nearest 0.1 of a minute.

[1 mark]

Answer ____________________ minutes

13 Use the substitution $u = x^4 + 2$ to find the value of $\int_0^1 \frac{x^7}{(x^4 + 2)^2}\, dx$

giving your answer in the form $p \ln q + r$ where p, q and r are rational numbers.

[6 marks]

Answer

END OF QUESTIONS

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