OXFORD

INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

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INTERNATIONAL A-LEVEL

MATHEMATICS

(9660/MA04) Unit S2 − Statistics

Specimen 2018 Morning Time allowed: 1 hour 30 minutes

Materials

- For this paper you must have the booklet of formulae and statistical tables.
- You may use a graphics calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use a supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box or around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions in the spaces provided.

1 The mean age of people attending a large concert is claimed to be 35 years.

A random sample of 100 people attending the concert was taken and the mean age was found to be 37.9 years.

1 (a) Given that the standard deviation of the ages of the people attending the concert is 12 years, test, at the 1% level of significance, the claim that the mean age is 35 years.

[7 marks]

1 (b) Explain, in context, the meaning of a Type II error.

[2 marks]

2 The continuous random variable T has probability density function $\mathrm{f}(t)$, where

$$\mathrm{f}(t) = \begin{cases} 5\mathrm{e}^{-5t} & t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

2 (a) Derive the cumulative distribution function of T.

[4 marks]

2 (b) Find the value of the constant c such that $\mathrm{P}(T > c) = 0.05$

[2 marks]

$c =$ _____

3 The weight, Y kilograms, of cement in a bag can be modelled by a normal random variable with mean 25.25 and standard deviation 0.35

A firm purchases 10 such bags. These bags may be considered to be a random sample.

3 (a) Determine the probability that the mean weight of cement in the 10 bags is less than 25 kg.

[4 marks]

Answer _____

3 (b) Calculate the probability that the weight of cement in each of the 10 bags is more than 25 kg.

[4 marks]

Answer _____

4 Jack, a geologist, is studying pebbles on a beach. He uses a frame, called a quadrat, to enclose an area of the beach. He then counts the number of quartz pebbles, X, within the quadrat. He repeats this procedure 40 times, obtaining the following summarised data:

$$\sum x = 128 \quad \text{and} \quad \sum (x - \bar{x})^2 = 126.4$$

Jack believes that the distribution of X can be modelled by a Poisson distribution with $\lambda = 3.2$

4 (a) Use the summarised data to support Jack's belief.

[3 marks]

4 (b) Using Jack's model, calculate the probability that

4 (b) (i) a single placing of the quadrat contains more than 5 quartz pebbles

[2 marks]

Answer _____

4 (b) (ii) a single placing of the quadrat contains at least 3 quartz pebbles but fewer than 8 quartz pebbles

[3 marks]

Answer _____

4 (b) (iii) when the quadrat is placed **twice**, both placings contain at most 5 quartz pebbles.

[2 marks]

Answer _____

4 (c) Jack models the number of **flint** pebbles enclosed by the quadrat by a Poisson distribution with mean 5. He assumes that the number of flint pebbles enclosed by the quadrat is independent of the number of quartz pebbles enclosed by the quadrat.

Using Jack's models, calculate the probability that a single placing of the quadrat contains a **total** of either 9 or 10 pebbles which are quartz or flint.

[3 marks]

Answer _____

5 It is claimed that the proportion, p, of people who prefer cooked fresh sweetcorn to cooked frozen sweetcorn is greater than 0.50

In an attempt to test this claim, a sample of 50 people were each given an unlabelled portion of cooked fresh sweetcorn and an unlabelled portion of cooked frozen sweetcorn to taste.

After tasting each portion, each person was asked to state which of the two portions they preferred.

Of the 50 people sampled, 29 preferred the cooked fresh sweetcorn.

Assuming that the 50 people may be considered to constitute a random sample, use a binomial distribution and the 10% level of significance to investigate the claim.

[6 marks]

6 The volume of shampoo, V millilitres, delivered by a machine into bottles may be modelled by a normal random variable with mean μ and standard deviation σ.

6 (a) Given that $\mu = 412$ and $\sigma = 8$, determine

6 (a) (i) P($V < 400$)

[3 marks]

Answer _____

6 (a) (ii) P($V > 420$)

[2 marks]

Answer _____

6 (a) (iii) P($V = 410$).

[1 mark]

Answer _____

6 (b) A new quality control specification requires that the values of μ and σ are changed so that

$$P(V < 400) = 0.05 \quad \text{and} \quad P(V > 420) = 0.01$$

6 (b) (i) Show, with the aid of a suitable sketch or otherwise, that

$$400 - \mu = -1.6449\sigma \quad \text{and} \quad 420 - \mu = 2.3263\sigma$$

[3 marks]

6 (b) (ii) Hence calculate values for μ and σ.

[3 marks]

$\mu =$ _____

$\sigma =$ _____

7 The time, in weeks, that a patient must wait to be given an appointment at Holmsoon Hospital may be modelled by a random variable T having the cumulative distribution function

$$\mathrm{F}(t) = \begin{cases} 0 & t \leq 0 \\ \dfrac{t^3}{216} & 0 \leq t \leq 6 \\ 1 & t \geq 6 \end{cases}$$

7 (a) Find, to the nearest day, the time within which 90 per cent of patients will have been given an appointment.

[3 marks]

Answer _____

7 (b) Find the probability density function of T for all values of t.

[3 marks]

7 (c) Calculate the mean and the variance of T.

[6 marks]

mean = _____

variance = _____

7 (d) Calculate the probability that the time that a patient must wait to be given an appointment is more than one standard deviation above the mean.

[4 marks]

Answer _____

8 The contents, in millilitres, of cartons of milk produced at Kream Dairies can be modelled by a normal distribution with mean 568 and variance σ^2

After receiving several complaints from their customers who thought that the average content of the cartons had been reduced, the production manager of Kream Dairies decided to investigate.

A random sample of 8 cartons of milk was taken, revealing the following contents, in millilitres.

560 568 561 562 564 567 565 563

Investigate, at the 1% level of significance, whether the average content of cartons of milk is less than 568 millilitres.

[10 marks]

END OF QUESTIONS

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