OXFORD
INTERNATIONAL
AQA EXAMINATIONS

INTERNATIONAL A-LEVEL

FURTHER MATHEMATICS

(9665)

Mark scheme

Further pure mathematics Unit 1

Specimen

Principal Examiners have prepared these mark schemes for specimen papers. These mark schemes have not, therefore, been through the normal process of standardising that would take place for live papers.

Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
ft	Follow through from previous incorrect result
CAO	Correct and answer only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
$-x$ **EE**	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

No method shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Answer	Marks	Total	Comments
1(a)	$\mathrm{f}(r+1)-\mathrm{f}(r)=r(r+1)^2-(r-1)r^2$	M1		
	$=r\left(r^2+2r+1-r^2+r\right)$	A1		any expanded form
	$=r(3r+1)$	A1	3	AG
(b)	$r=50 \quad \mathrm{f}(51)-\mathrm{f}(50)$ $r=51 \quad \mathrm{f}(52)-\mathrm{f}(51)$ PI $r=99 \quad \mathrm{f}(100)-\mathrm{f}(99)$	M1A1		OE clearly shown. Accept $\sum_{1}^{99}-\sum_{1}^{49}$
	$\sum_{r=50}^{99} r(3r+1)=\mathrm{f}(100)-\mathrm{f}(50)$	m1		clear cancellation
	$=867500$	A1	4	cao
		Total	7	

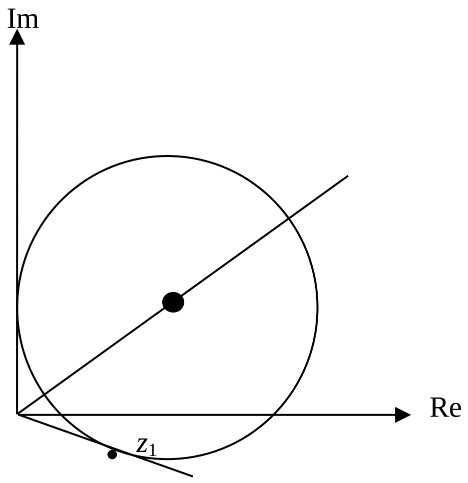
Q	Answer	Marks	Total	Comments
2(a)	$\alpha+\beta=-\frac{7}{2}$	B1		
	$\alpha\beta=4$	B1	2	
(b)	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=\left(-\frac{7}{2}\right)^2-2(4)$	M1		Using correct identity with ft or correct substitution
	$=\frac{49}{4}-8=\frac{17}{4}$	A1	2	CSO AG. A0 if $\alpha+\beta$ has wrong sign

(c)	(Sum =) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{\alpha^2+\beta^2}{(\alpha\beta)^2}=\frac{17/4}{16}\left(=\frac{17}{64}\right)$	M1		Writing $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$ in a correct suitable form with ft or correct substitution
	$=\frac{17}{64}$	A1ft		ft wrong value for $\alpha\beta$
	(Product =) $\frac{1}{(\alpha\beta)^2}=\frac{1}{16}\left(=\frac{4}{64}\right)$	B1ft		ft wrong value for $\alpha\beta$
	x^2-Sx+P (=0)	M1		Using correct general form of LHS of eqn with ft substitution of their S and P values. PI
	Eqn is $64x^2 - 17x + 4 = 0$	A1	5	CSO Integer coefficients and '= 0' needed
		Total	9	

Q	Answer	Marks	Total	Comments
3(a)	$\int 2x^{-3}\mathrm{d}x=-x^{-2}(+c)$	M1A1		M1 for correct index
	$\int_p^q 2x^{-3}\mathrm{d}x=p^{-2}-q^{-2}$	A1ft	3	OE; ft wrong coefficient of x^{-2}
(b)(i)	As $p\to 0$, $p^{-2}\to\infty$, so no value	B1		
(ii)	As $q\to\infty$, $q^{-2}\to 0$, so value is ¼	M1A1ft	3	ft wrong coefficient of x^{-2}
		Total	6	
4 (a)	Use of $z^* = x - \mathrm{i}y$	M1		
	$(z - \mathrm{i})(z^* - \mathrm{i}) = (x^2 + y^2 - 1) - 2\mathrm{i}x$	m1A1	3	A1 may be earned in **(b)**
(b)	Equating R and I parts			
	$-2x = -8$ so $x = 4$	M1 A1		
	$16 + y^2 - 1 = 24$ so $y=\pm 3\,(z=4\pm 3\mathrm{i})$	m1A1	4	A0 if $x = -4$ used
		Total	7	

5(a)	$(5+h)^3 = 125+75h+15h^2+h^3$	B1	1	Accept unsimplified coefficients
(b)(i)	$y(5+h) = 100+65h+14h^2+h^3$ Use of correct formula for gradient Gradient is $65+14h+h^2$	B1ft M1 A2,1ft	4	PI; ft numerical error in **(a)** A1 if one numerical error made; ft numerical error already penalised
(ii)	As $h \to 0$ this $\to 65$	E2,1ft	2	E1 for 'h = 0'; ft wrong values for p, q, r
		Total	7	

Q	Answer	Marks	Total	Comments
6(a)	$\sum r^2(4r-3) = 4\sum r^3 - 3\sum r^2 \ldots$	M1		Splitting up the sum into two separate sums. PI by next line.
	$= 4\left(\frac{1}{4}\right)n^2(n+1)^2 - 3\left(\frac{1}{6}\right)n(n+1)(2n+1)$	m1		Substitution of the two summations from FB
	$= n\left(n+1\right)\left[n\left(n+1\right) - \frac{1}{2}\left(2n+1\right)\right]$	m1		Taking out common factors n and $n+1$.
		A1		Remaining expression eg our […] in ACF not just simplified to AG
	Sum = $\frac{1}{2}n(n+1)(2n^2-1)$	A1	5	Be convinced as form of answer is given, penalise any jumps or backward steps
(b)	$\sum_{r=20}^{40} r^2(4r-3)$			
	$= \sum_{r=1}^{40} r^2(4r-3) - \sum_{r=1}^{19} r^2(4r-3)$	M1		Attempt to take S(19) from S(40) using part **(a)**
	= 20(41)(3199) − 9.5(20)(721) = 2623180 – 136990			
	$\sum_{r=20}^{40} r^2(4r-3) = 2486190$	A1	2	2486190 ; Since ‘Hence’ NMS 0/2. SC $\sum_{r=1}^{40}$....... − $\sum_{r=1}^{20}$...... clearly attempted and evaluated to 2455390 scores B1
		Total	7	

Q	Answer	Marks	Total	Comments
7(a)	 Half-line with gradient < 1	B1	1	condone a short line, ie it stops at or inside circle
(b)(i)	Circle centre on L, x-coord 6 indicated touching Re(z) = 0 not at (0, 0)	B1 B1	2	 not touching Re axis
(ii)	y-coord of centre is $2\sqrt{3}$ or $\frac{6}{\sqrt{3}}$ $z_0 = 6 + 2\sqrt{3}\,\mathrm{i}$, k = 6	B1 B1ft B1	3	OE; PI ft error in coords of centre
(iii)	Point z_1 shown $\arg z_1 = -\frac{1}{6}\pi$	B1 B1	2	PI
		Total	8	

Q	Answer	Marks	Total	Comments
8	$\sin\left(-\frac{\pi}{6}\right)=-\frac{1}{2}$	B1		OE; dec/deg penalised at 6th mark
	$\sin\left(-\frac{5\pi}{6}\right)=-\frac{1}{2}$	B1ft		OE; ft wrong first value
	Use of $2n\pi$	M1		
	Going from $4x-\frac{2\pi}{3}$ to x □	m1		(or $n\pi$) at any stage including division of all terms by 4
	GS $x=\frac{\pi}{8}+\frac{1}{2}n\pi$ or $x=-\frac{\pi}{24}+\frac{1}{2}n\pi$	A1A1ft	6	OE; ft wrong first or second value
		Total	6	

Q	Answer	Marks	Total	Comments
9(a)(i)	Asymptotes x = 3 and y = 0	B1,B1	2	may appear on graph
(ii)	Complete graph with correct shape Coordinates $\left(0,-\frac{1}{3}\right)$ shown	B1 B1	2	
(iii)	Correct line, (0, −5) and (2.5, 0) shown	B1	1	
(b)(i)	$2x^2-11x+14=0$	B1		
	x = 2 or x = 3.5	M1A1	3	M1 for valid method for quadratic
(ii)	$2 < x < 3$, $x > 3.5$	B2,1ft	2	B1 for partially correct solution; ft incorrect roots of quadratic (one above 3, one below 3)
		Total	10	

Q	Answer	Marks	Total	Comments
10(a)(i)	Parabola drawn passing through (2, 0)	M1 A1	 2	with x-axis as line of symmetry
(ii)	Two tangents passing through (−2, 0)	B1B1	2	to their parabola
(b)(i)	Elimination of y Correct expansion of $(x+2)^2$ Result	M1 B1 A1	 3	 convincingly shown (AG)
(ii)	Correct discriminant $16m^4 - 8m^2 + 1 = 16m^4 + 8m^2$ Result	B1 M1 A1	 3	 OE convincingly shown (AG)
(iii)	$\frac{1}{16}x^2 - \frac{3}{4}x + \frac{9}{4} = 0$ $x = 6$, $y = \pm 2$	M1 A1A1	 3	OE
		Total	13	
		Total	**80**	

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