OXFORD
INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________

INTERNATIONAL AS
FURTHER MATHEMATICS

(FM01) Further Pure Mathematics Unit 1

Specimen 2018   Morning   Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the booklet of formulae and statistical tables.
- You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use a supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box or around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions in the spaces provided.

1(a) Given that $f(r) = (r - 1)r^2$, show that

$$f(r + 1) - f(r) = r(3r + 1)$$

[3 marks]

(b) Use the method of differences to find the value of

$$\sum_{r=50}^{99} r(3r+1)$$

[4 marks]

Answer

2 The quadratic equation

$$2x^2 + 7x + 8 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$.

[2 marks]

$\alpha + \beta =$ __________

$\alpha\beta =$ __________

(b) Show that $\alpha^2 + \beta^2 = \dfrac{17}{4}$

[2 marks]

(c) Find a quadratic equation, with integer coefficients, which has roots

$$\frac{1}{\alpha^2} \text{ and } \frac{1}{\beta^2}$$

[5 marks]

Answer

3(a) Find, in terms of p and q, the value of the integral $\int_p^q \frac{2}{x^3}\,dx$.

[3 marks]

Answer

(b) Show that only **one** of the following improper integrals has a finite value, and find that value:

(i) $\int_0^2 \frac{2}{x^3}\,dx$

(ii) $\int_2^\infty \frac{2}{x^3}\,dx$

[3 marks]

4 It is given that $z = x + \mathrm{i}y$, where x and y are real.

(a) Find, in terms of x and y, the real and imaginary parts of

$$(z - \mathrm{i})(z^* - \mathrm{i})$$

[3 marks]

Answer

(b) Given that

$$(z - \mathrm{i})(z^* - \mathrm{i}) = 24 - 8\,\mathrm{i}$$

find the two possible values of z.

[4 marks]

$z =$

$z =$

5 (a) Expand $(5 + h)^3$

[1 mark]

Answer

(b) A curve has equation $y = x^3 - x^2$

(i) Find the gradient of the line passing through the point (5, 100) and the point on the curve for which $x = 5 + h$. Give your answer in the form

$$p + qh + rh^2$$

where p, q and r are integers.

[4 marks]

Answer

(b) (ii) Show how the answer to part **(b)(i)** can be used to find the gradient of the curve at the point (5, 100). State the value of this gradient.

[2 marks]

gradient =

6 (a) Use the formulae for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$ to show that

$$\sum_{r=1}^{n} r^2(4r-3) = kn(n+1)(2n^2-1)$$

where k is a constant.

[5 marks]

(b) Hence evaluate

$$\sum_{r=20}^{40} r^2(4r-3)$$

[2 marks]

Answer ____________________

7 (a) Draw on an Argand diagram the locus L of points satisfying the equation $\arg z = \dfrac{\pi}{6}$.

[1 mark]

(b) (i) A circle C of radius 6 has its centre lying on L and touches the line $\mathrm{Re}(z) = 0$.
Draw C on the same Argand diagram.

[2 marks]

(b) (ii) Find the equation of C, giving your answer in the form $|z - z_0| = k$.

[3 marks]

Answer ______________________

(b) (iii) The complex number z_1 lies on C and is such that arg z_1 has its least possible value.
Find arg z_1, giving your answer in the form $p\pi$, where $-1 < p \leq 1$

[2 marks]

Answer ______________________

8 Find the general solution of the equation

$$\sin\left(4x - \frac{2\pi}{3}\right) = -\frac{1}{2}$$

giving your answer in terms of π.

[6 marks]

Answer ______________________

9 (a) (i) Write down the equations of the **two** asymptotes of the curve $y = \dfrac{1}{x-3}$.

[2 marks]

(a) (ii) Sketch the curve $y = \dfrac{1}{x-3}$, showing the coordinates of any points of intersection with the coordinate axes.

[2 marks]

(a) (iii) On the same axes, again showing the coordinates of points of intersection with the coordinate axes, sketch the line $y = 2x - 5$.

[1 mark]

(b) (i) Solve the equation

$$\frac{1}{x-3} = 2x - 5$$

[3 marks]

Answer ______________________

(b) (ii) Find the solution of the inequality

$$\frac{1}{x-3} < 2x - 5$$

[2 marks]

Answer ______________________

10 A parabola *P* has equation $y^2 = x - 2$

(a) (i) Sketch the parabola *P*.

[2 marks]

(a) (ii) On your sketch, draw the **two** tangents to *P* which pass through the point (–2, 0).

[2 marks]

(b) (i) Show that, if the line $y = m(x+2)$ intersects *P*, then the x-coordinates of the points of intersection must satisfy the equation

$$m^2x^2 + (4m^2 - 1)x + (4m^2 + 2) = 0$$

[3 marks]

(b) (ii) Show that, if this equation has equal roots, then

$$16m^2 = 1$$

[3 marks]

(b) (iii) Hence find the coordinates of the points at which the tangents to *P* from the point (–2, 0) touch the parabola *P*.

[3 marks]

END OF QUESTIONS

There are no questions printed on this page

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