OXFORD
INTERNATIONAL
AQA EXAMINATIONS

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS

(9665)

Mark scheme

Further pure statistics and mechanics Unit 1

Specimen

Principal Examiners have prepared these mark schemes for specimen papers. These mark schemes have not, therefore, been through the normal process of standardising that would take place for live papers.

Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
OE	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
$-x$ **EE**	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

No method shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

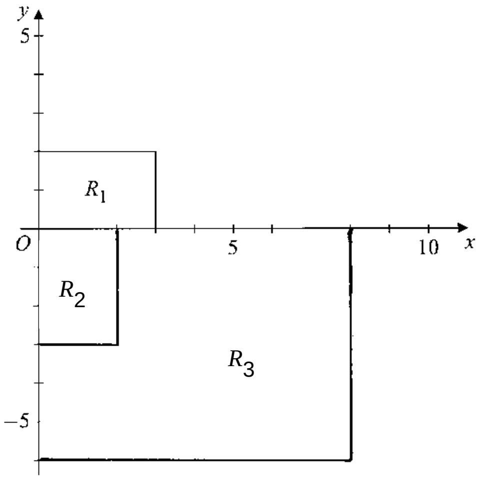
Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Answer	Marks	Comments
1	$h\,y'(9) = 0.25 \times \dfrac{1}{2+\sqrt{9}}$ (= 0.05)	M1	Attempt to find $h\,y'(9)$.
	$\{y(9.25)\} \approx 6 + 0.05 = 6.05$	A1	6.05 OE
	$\{y(9.5)\} \approx y(9.25) + 0.25 \times y'(9.25)$ $\approx 6.05 + 0.25 \times \dfrac{1}{2+\sqrt{9.25}}$ $\approx 6.05 + 0.25 \times 0.1983(5\ldots)$	m1	Attempt to find $y(9.25) + 0.25 \times y'(9.25)$, must see evidence of numerical expression if correct ft [0.049(5..)+c's $y(9.25)$] value is not obtained.
	$\approx 6.05 + 0.0495(8\ldots..)$	A1ft	PI; ft on their value for $y(9.25)$; 4 dp value (rounded or truncated) or better.
	$y(9.5) = 6.0996$ (to 4 d.p.)	A1	$y(9.5) = 6.0996$
	Total	5	

In this question, misreads lose all the A marks that are affected

Q	Answer	Marks	Comments
2(a)(i)		B1	Rectangle with vertices (0, 0), (0, −3), (2, −3), (2, 0)
2(a)(ii)		M1 A2,1	Rectangle with vertices **either** whose x-coords are their **(a)(i)** x-coord vertices multiplied by 4 **or** whose y-coords are their **(a)(i)** y-coord vertices multiplied by 2 A2 if rectangle with vertices (0, 0), (0, −6), (8, −6), (8, 0) (A1 if either the four x-coords are correct or the four y-coords are correct)
2(b)(i)	Matrix is $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$	B1	
2(b)(ii)	$\begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} =$ $\begin{bmatrix} 0 & 4 \\ -2 & 0 \end{bmatrix}$	M1 m1 A1	Attempt to multiply $\begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$ with their **(b)(i)** matrix in **either** order. Multiplication in **correct** order with at least two of the four ft multiplications carried out correctly. For $\begin{bmatrix} 0 & 4 \\ -2 & 0 \end{bmatrix}$ NMS $\begin{bmatrix} 0 & 4 \\ -2 & 0 \end{bmatrix}$ scores B3 $\begin{bmatrix} 0 & 2 \\ -4 & 0 \end{bmatrix}$ scores B1
	Total	**8**	

Q	Answer	Marks	Comments
3(a)	Use of one law of logs or exponentials log $a = c$ and log $b = m$ So $a = 10^c$ and $b = 10^m$	M1 A1 A1	 OE, both needed
3(b)	Points (1, 1.08), (5, 1.43) plotted Straight line drawn through points	M1A1 A1ft	M1A0 if one point correct ft small inaccuracy
3(c)(i)	Attempt at antilog of $Y(3)$ When $x = 3$, $Y \approx 1.25$ so $y \approx 18$	M1 A1	OE Allow AWRT 18
3(c)(ii)	Attempt at a as antilog of Y-intercept $a \approx 9.3$ to 10	M1 A1	OE AWRT
	Total	**10**	

4(a)	Determinant of matrix = –8 + 9 = 1 Area = 3 $\times$ 1 = 3 (square units)	M1 A1	Finding determinant and multiplying by area CAO must show multiplication or refer to scale factor/invariant area or equivalent
4(b)(i)	$\begin{bmatrix} 4 & 3 \\ -3 & -2 \end{bmatrix}\begin{bmatrix} x \\ mx+c \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$ $\Rightarrow$ $(x') = 4x + 3(mx+c)$ $(y') = -3x - 2(mx+c)$ Invariant lines $\Rightarrow y' = mx' + c$ $\Rightarrow -3x - 2mx - 2c = 4mx + 3m^2x + 3mc + c$ $\Rightarrow 0 = \left(3m^2 + 6m + 3\right)x + 3mc + 3c$	M1 A1	x', y' in terms of x, y, m, c Use of $y' = mx' + c$
	$\Rightarrow 3m^2 + 6m + 3 = 0 \quad 3mc + 3c = 0$ $3\left(m+1\right)^2 = 0 \quad 3c\left(m+1\right) = 0$ $\Rightarrow m = -1 \quad c$ can be any value $\Rightarrow$ lines are $y = -x + c$	M1 A1 A1	Attempt at solving equations where coefficients = 0 or compares coefficients Finding the correct value of m Fully correct line – no restriction on c

Q	Answer	Marks	Comments
4(b)(ii)	When $c = 0$, $y = -x$ is a line of invariant points **SPECIAL CASES – (b)(i)** $\begin{pmatrix} 4 & 3 \\ -3 & -2 \end{pmatrix}\begin{pmatrix} x \\ -x+c \end{pmatrix} = \begin{pmatrix} x+3c \\ -x-2c \end{pmatrix}$ $x' = x + 3c$ $y' = -x - 2c$ Consider $-x' + c$ $= -(x+3c) + c$ $= -x - 3c + c$ $= -x - 2c$ $= y'$ Hence $y = -x + c$ is an invariant line	B1	Any equivalent form SC1 Correct multiplication as shown SC2 correct multiplication as shown above and full algebraic solution using $y' = -x' + c$
		Total	**8**

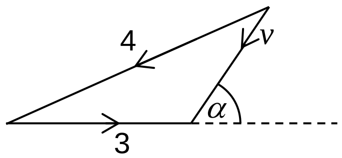
Q	Answer	Marks	Comments
5(a)	Let $f(x) = 24x^3 + 36x^2 + 18x - 5$ $f(0.1) = -2.816$, $f(0.2) = 0.232$	M1	Both attempted and at least one evaluated correctly to at least 1sf rounded or truncated OE fraction
	Change of sign so α lies between 0.1 and 0.2	A1	Need both evaluations correct to above degree of accuracy and 'change of sign OE' and relevant reference to 0.1 and 0.2
5(b)	$f(0.15) = -1.409$ (< 0 so root > 0.15)	M1	$f(0.15)$ considered first
	$f(0.175) \approx -0.619$ (< 0 so root > 0.175)	A1	$f(0.15)$ then $f(0.175)$ both evaluated correctly to at least 1sf OE fractions
	α lies between 0.175 and 0.2	A1	Dependent on both previous marks gained and no other additional evaluations other than at 0.15 and 0.175
5(c)	$f'(x) = 72x^2 + 72x + 18$	B1	PI
	$(x_2 =)$ $0.2 - \dfrac{24(0.2)^3 + 36(0.2)^2 + 18(0.2) - 5}{72(0.2)^2 + 72(0.2) + 18}$	B1 B1	B1 for numerator in correct formula B1 for denominator in correct formula
	$= 0.1934$ (to 4dp)	B1	CAO Must be 0.1934 Do not apply ISW NMS scores 0/4
	Total	**9**	

Q	Answer	Marks	Comments
6(a)	P(U) = (0.40 × 0.15) + (0.45 × 0.05) + (0.15 × 0.10)	M1	≥ 2 terms correct; may be implied
	= 0.06 + 0.0225 + 0.015 = **0.097 to 0.098**	A1	AWFW (0.0975)
6(b)	$P(D \mid U) = \frac{P(D \cap U)}{P(U)} = \frac{0.40 \times 0.15}{\text{their } \textbf{(a)}}$	M1	May be implied
	$= \frac{0.06}{0.0975}$ = **0.612 to 0.619**	A1	AWFW (0.61538)
6(c)	$P(S \mid O) = \frac{0.15 \times (1 - 0.10)}{1 - \text{their } \textbf{(a)}} = \frac{0.135}{0.9025}$	M1 M1	Numerator Denominator
	= **0.149 to 0.15**	A1	AWFW (0.14958)
	Total	**7**	

Q	Answer	Marks	Comments
7(a)	$\text{E}(X) = \sum x\text{P}(X = x)$		
	$= \sum_{x=1}^{3n} \frac{x}{3n} = \frac{1}{3n}\sum_{x=1}^{3n} x$	M1	Ignore limits
	$= \frac{1}{3n}\left(\frac{3n(3n+1)}{2}\right)$	m1	Sum of first $3n$ integers
	$= \frac{3n+1}{2}$	A1	Fully correct and complete derivation
7(b)	$\text{E}(X) = 14$ and $\text{Var}(x) = \frac{728}{12} = \frac{182}{3}$ $= 60.\dot{6}$	B1	Both CAO and CAO/AWRT oe
	$\text{E}(X) + \sqrt{\text{Var}(X)} = 14 + 7.7\dot{8} = 21.8$	M1	Use of $\text{E}(X) + \sqrt{\text{Var}(X)}$
	$\text{P}(X < 21.8) = \frac{21}{27}$ or $\frac{7}{9}$	A1	CAO
	Total	**6**	

Q	Answer	Marks	Comments
8(a)	$G_u(t) = E(t^u) = \sum t^u \binom{n}{u} p^u q^{n-u}$	M1	Allow x instead of u and no definition of q
	$q = 1 - p$		
	$= \sum_{u=0}^{n} \binom{n}{u} (pt)^u \; q^{n-u}$	m1	Combining terms and correct limits
	$= (q + pt)^n$	A1	Fully correct and complete derivation
8(b)(i)	$G_W(t) = (q + pt)^{2n}$	B1	
	$G_W(t) = G_U(t) \times G_V(t)$	M1	Used
	$= (q + pt)^n \times (q + pt)^{2n} = (q + pt)^{3n}$	A1	
8(b)(ii)	$W \sim B(3n, p)$	B1	oe
	Total	**7**	

Q	Answer	Marks	Comments
9(a)	$\tan\alpha = \frac{4}{3}$ or $\cos\alpha = \frac{3}{5}$ or $\sin\alpha = \frac{4}{5}$ $\alpha = 53.1°$ AG	M1 A1	M1 Trig equation to find the angle with: cos with 3 or 4 in the numerator and 5 in denominator sin with 3 or 4 in the numerator and 5 in denominator tan with 3 and 4 in any position A1: Correct angle from correct working. (Allow 90 – 36.9 = 53.1°). Final answer must be 53.1 Note, for example, $\tan^{-1}\frac{4}{3} = 53.1$ scores M1A1

Q	Answer	Marks	Comments
9(b)		M1 A1	(Note diagram not needed for the award of marks)
	$4^2 = 3^2 + v^2 - 2\times 3\times v\times\cos(180-53.1...)$ $v^2 + 3.6v - 7 = 0$ $v = 1.40$ or $v = -5.00$ $v = 1.40\ \text{m s}^{-1}$	B1 M1A1 A1 m1 A1	B1 For seeing 180 – 53.1 (= 126.9). M1 Using cosine rule with 3, 4, v and any angle. Must see v **and** v^2. A1 Correct equation. A1 Correct simplified quadratic. m1 Solving the quadratic. A1 Selecting positive root. (Can be implied.) Accept 1.4 or 1.39
	OR		
	$\dfrac{\sin(180-53.13)}{4} = \dfrac{\sin\theta}{3}$ $\theta = 36.87°$ $180 - 36.87 - 126.87 = 16.26°$ $\dfrac{v}{\sin 16.26°} = \dfrac{4}{\sin(180-53.13)}$ OR $\dfrac{3}{\sin 36.87°}$ $v = 1.40\ \text{m s}^{-1}$	(B1) (M1A1) (A1) (m1) (A1)	B1 For seeing 180 – 53.1 (= 126.9). M1 Using sine rule with 3, 4 and 126.9º. A1 Correct equation. A1 For finding 16.26. Accept 16.3 or 16.2 or 16.26... . m1 Second application of sine rule with v and 3 or 4 with at least one correct angle. A1 Correct velocity. Accept 1.4 or 1.39. Note: the result below can be proved. $v = 4\sin\alpha - 3\cos\alpha$ SC4 seeing $4\sin\alpha - 3\cos\alpha$ with incorrect answer. SC6 seeing $4\sin\alpha - 3\cos\alpha$ with answer as 1.4 or 1.39.
	Total	**8**	

Q	Answer	Marks	Comments
10	Dimension of g is $\mathrm{LT^{-2}}$ Dimension of s is L Dimension of h is L Dimension of m_1 and m_2 is M	B1	B1 for dimensions of the five quantities
	Dimension of $\frac{g}{s}[s(m_1+m_2)+\frac{hm_1^2}{m_1+m_2}]$ is		
	$\frac{\mathrm{LT^{-2}}}{\mathrm{L}}[\mathrm{LM}+\frac{\mathrm{LM^2}}{\mathrm{M}}] \cong \mathrm{MLT^{-2}}+\mathrm{MLT^{-2}}$	M1	Correct substitution of dimension
	$\cong \mathrm{MLT^{-2}}$	A1	
	which is a force	B1	
	Total	4	

Q	Answer	Marks	Comments
11(a)	$m(4u)+3m(2u)=mv_A+3mv_B$	M1 A1	M1 for four correct momentum terms with any signs. A1 for all correct
	$\dfrac{v_B-v_A}{4u-2u}=e$	M1 A1	M1 for correct terms for any signs, A1 for all correct.
	$\left(\begin{array}{l} v_A+3v_B=10u \\ v_B-v_A=2ue \\ 4v_B=2ue+10u \end{array}\right)$		
	$v_B=\dfrac{u}{2}(e+5)$	A1	OE, simplified
	$\left(v_A=\dfrac{u}{2}(e+5)-2ue\right)$		
	$v_A=\dfrac{u}{2}(-3e+5)$	A1	OE, simplified
11(b)	$e\le 1 \quad \Rightarrow \quad v_B\le\dfrac{u}{2}(1+5)$	M1	Use of $e\le 1$ (OE) needed
	$\Rightarrow \quad v_B\le 3u$	A1	FT their v_B
	Total	**8**	

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