OXFORD
INTERNATIONAL AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ______________________

Forename(s) ______________________

Candidate signature ______________________

INTERNATIONAL AS
FURTHER MATHEMATICS

(FM02) Further Pure, Statistics and Mechanics Unit 1

Specimen 2018 Morning Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the booklet of formulae and statistical tables.
- You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use a supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box or around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- Unless otherwise stated, use $g = 9.8$ ms^{-2}

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions in the spaces provided.

1 A curve passes through the point (9, 6) and satisfies the differential equation

$$\frac{dy}{dx} = \frac{1}{2 + \sqrt{x}}$$

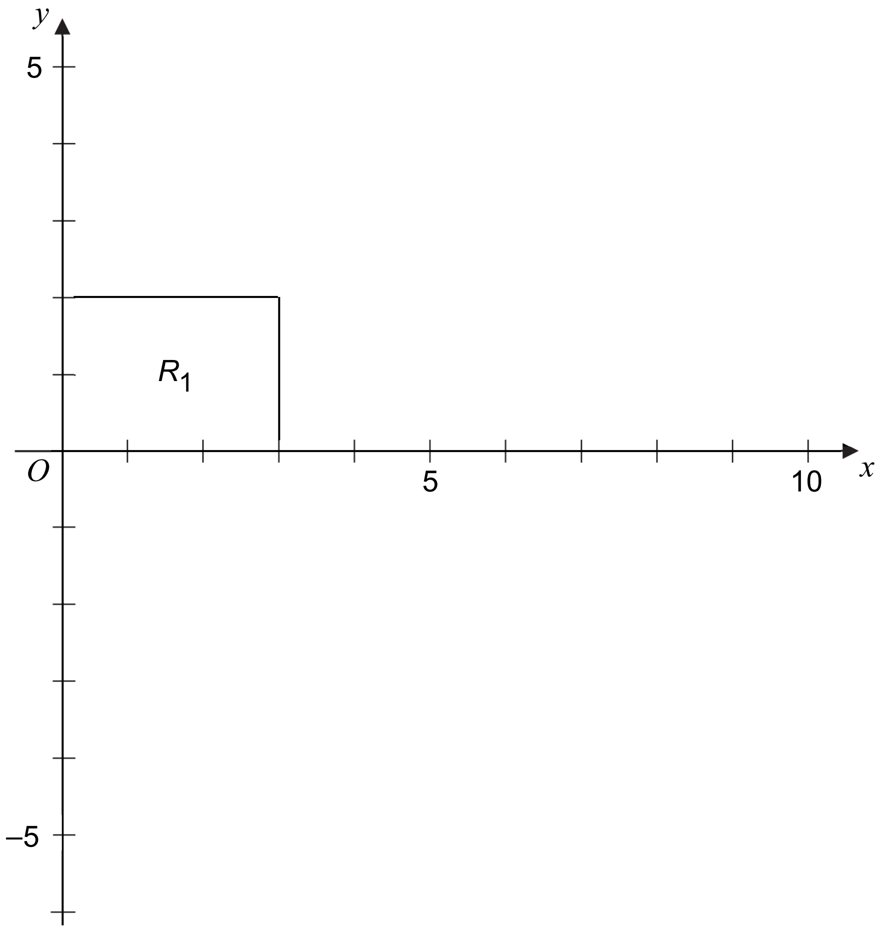
Use a step-by-step method with a step length of 0.25 to estimate the value of y at $x = 9.5$

Give your answer to four decimal places.

[5 marks]

Answer ______________________

2 The diagram below shows a rectangle R_1 which has vertices (0, 0), (3, 0), (3, 2) and (0, 2).



2 (a) On the diagram, draw

2 (a) (i) the image R_2 of R_1 under a rotation through 90° clockwise about the origin

[1 mark]

2 (a) (ii) the image R_3 of R_2 under the transformation which has matrix

$$\begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$$

[3 marks]

2 (b) Find the matrix of:

2 (b) (i) the rotation which maps R_1 onto R_2

[1 mark]

Answer

2 (b) (ii) the combined transformation which maps R_1 onto R_3

[3 marks]

Answer

3 The variables x and Y, where $Y = \log_{10} y$, are related by the equation

$$Y = mx + c$$

where m and c are constants.

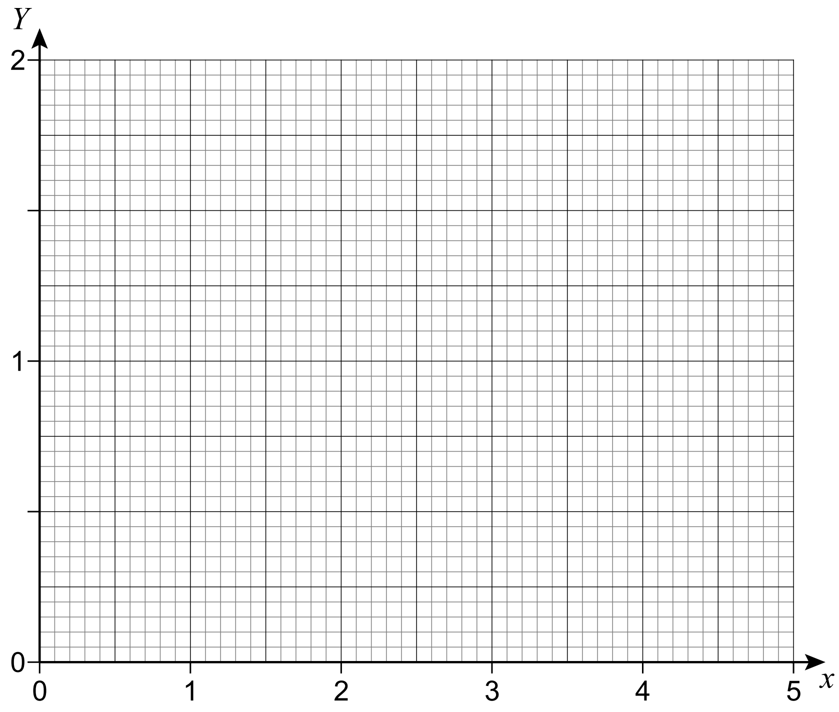
3 (a) Given that $y = ab^x$, express a in terms of c, and b in terms of m.

[3 marks]

3 (b) It is given that $y = 12$ when $x = 1$ and that $y = 27$ when $x = 5$

On the diagram opposite, draw a linear graph relating x and Y.

[3 marks]



3 (c) Use your graph to estimate, to two significant figures:

3 (c)(i) the value of y when $x = 3$;

[2 marks]

Answer ______________________

3 (c) (ii) the value of a.

[2 marks]

Answer ______________________

4 The plane transformation T is defined by

$$\mathrm{T} : \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

4 (a) A shape has an area of 3 square units.

Find the area of the shape after being transformed by T.

[2 marks]

Answer ______________________

4 (b) (i) Find the equations of all the invariant lines of T.

[5 marks]

4 (b) (ii) State the equation of the line of invariant points of T.

[1 mark]

Answer

5 The equation $24x^3 + 36x^2 + 18x - 5 = 0$ has one real root, α

5 (a) Show that α lies in the interval $0.1 < x < 0.2$

[2 marks]

5 (b) Starting from the interval $0.1 < x < 0.2$, use interval bisection **twice** to obtain an interval of width 0.025 within which α must lie.

[3 marks]

Answer ______________________

5 (c) Taking $x_1 = 0.2$ as a first approximation to α, use the Newton-Raphson method to find a second approximation, x_2, to α.

Give your answer to four decimal places.

[4 marks]

Answer

6 A hotel has three types of room: double, twin and suite. The **percentage** of rooms in the hotel of each type is 40, 45 and 15 respectively.

Each room in the hotel may be occupied by 0, 1, 2 or 3 or more people.

The **proportional** occupancy of **each** type of room is shown in the table.

		Occupancy			
		0	**1**	**2**	**3 or more**
Room	**Double**	0.15	0.35	0.45	0.05
	Twin	0.05	0.55	0.30	0.10
	Suite	0.10	0.20	0.55	0.15

For example, the probability that, on a particular night, a double room has exactly 2 occupants is 0.45

On a particular night, a room is selected at random. Find the probability this room is

6 (a) unoccupied

[2 marks]

Answer ______________________

6 (b) a double room, given that it is unoccupied

[2 marks]

Answer ______________________

6 (c) a suite, given that it is occupied.

[3 marks]

Answer ______________________

7 A random variable X has the probability function

$$P(X = x) = \begin{cases} \dfrac{1}{3n} & x = 1, 2, 3, ..., 3n \\ 0 & \text{otherwise} \end{cases}$$

where n is a positive integer.

7 (a) Determine, in terms of n, an expression for $E(X)$.

[3 marks]

Answer ______________________

7 (b) Given that $\text{Var}(X) = \dfrac{9n^2 - 1}{12}$ and $n = 9$, calculate the exact value of

$\text{P}\left(X < (\text{E}(X) + \sqrt{\text{Var}(X)})\right)$

[3 marks]

Answer ______________________

8 The random variable U has a binominal distribution with parameters n and p.

8 (a) Derive the probability generating function, $G_u(t)$, of U.

[3 marks]

Answer ______________________________

8 (b) The random variable V is independent of U and has the distribution B($2n$, p)

You are given that $W = U + V$

8 (b) (i) Deduce an expression for $G_W(t)$;

[3 marks]

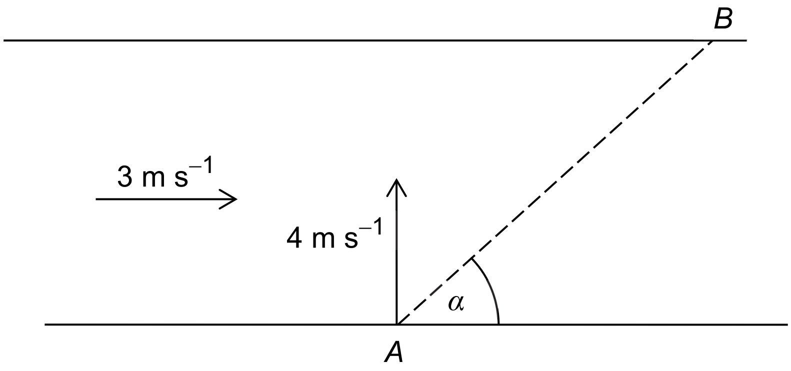
Answer ______________________________

8 (b) (ii) Hence specify the distribution of W.

[1 mark]

Answer

9 A river has straight parallel banks. The water in the river is flowing at a constant velocity of 3 m s^{-1} parallel to the banks. A boat crosses the river, from the point *A* to the point *B*, so that its path is at an angle α to the bank. The velocity of the boat relative to the water is 4 m s^{-1} perpendicular to the bank. The diagram shows these velocities and the path of the boat.



9 (a) Show that $\alpha = 53.1°$, correct to three significant figures.

[2 marks]

9 (b) The boat returns along the same straight path from B to A. Given that the speed of the boat relative to the water is still 4 m s^{-1}, find the magnitude of the resultant velocity of the boat on the return journey.

[6 marks]

Answer ____________________ m s^{-1}

10 A pile driver of mass M falls from a height h onto a pile of mass m, driving the pile a distance s into the ground. The pile driver remains in contact with the pile after the impact. A resistance force R opposes the motion of the pile into the ground.

Elizabeth finds an expression for R as

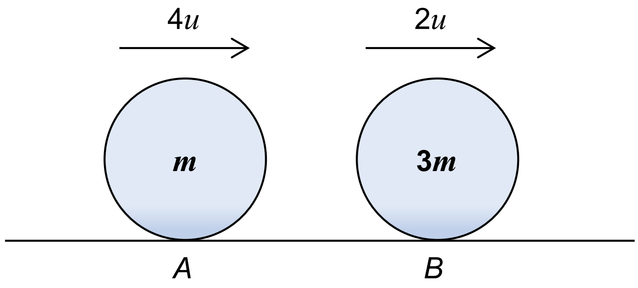
$$R = \frac{g}{s}\left[s(M + m) + \frac{hM^2}{M + m}\right]$$

where g is the acceleration due to gravity.

Determine whether the expression is dimensionally consistent.

[4 marks]

11 A smooth sphere *A*, of mass m, is moving with speed $4u$ in a straight line on a smooth horizontal table. A smooth sphere *B*, of mass $3m$, has the same radius as *A* and is moving on the table with speed $2u$ in the same direction as *A*.



The sphere *A* collides directly with sphere *B*.

The coefficient of restitution between *A* and *B* is e.

11 (a) Find, in terms of u and e, the speeds of *A* and *B* immediately after the collision.

[6 marks]

Speed of *A* =

Speed of *B* =

11 (b) Show that the speed of B after the collision cannot be greater than $3u$.

[2 marks]

END OF QUESTIONS

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