OXFORD
INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number						Candidate number				

Surname ______________________________

Forename(s) ______________________________

Candidate signature ______________________________

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS

(FM03) Unit FP2 − Pure Maths

Specimen 2018 Morning Time allowed: 2 hours 30 minutes

Materials
- For this paper you must have the booklet of formulae and statistical tables.
- You may use a graphics calculator.

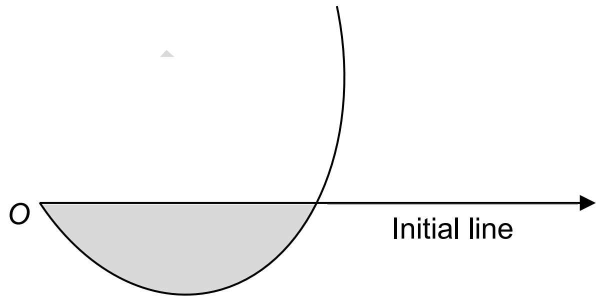
Instructions
- Use black ink or black ball-point pen. Pencil should be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use a supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box or around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 120.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions in the spaces provided.

1 The diagram shows a sketch of a curve C, the pole O and the initial line.



The polar equation of C is

$$r = 2\sqrt{1 + \tan\theta}\,, \quad -\frac{\pi}{4} \leqslant \theta \leqslant \frac{\pi}{4}$$

Show that the area of the shaded region, bounded by the curve C and the initial line, is $\frac{\pi}{2} - \ln 2$

[4 marks]

Turn over for the next question

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2 (a) Use the definitions of cosh θ and sinh θ in terms of e^{θ} to show that

$$\cosh x \cosh y - \sinh x \sinh y = \cosh (x - y)$$

[4 marks]

2 (b) It is given that x satisfies the equation

$$\cosh(x - \ln 2) = \sinh x$$

2 (b) (i) Show that $\tanh x = \frac{5}{7}$

[4 marks]

2 (b) (ii) Express x in the form $\frac{1}{2}\ln a$

[2 marks]

3 The roots of the cubic equation

$$z^3 - 2z^2 + pz + 10 = 0$$

are α, β and γ.

It is given that $\alpha^3 + \beta^3 + \gamma^3 = -4$

3 (a) Write down the value of $\alpha + \beta + \gamma$

[1 mark]

Answer

3 (b) (i) Explain why $\alpha^3 - 2\alpha^2 + p\alpha + 10 = 0$

[1 mark]

3 (b) (ii) Hence show that $\alpha^2 + \beta^2 + \gamma^2 = p + 13$

[4 marks]

3 (b) (iii) Deduce that $p = -3$

[2 marks]

3 (c) (i) Find the real root α of the cubic equation $z^3 - 2z^2 - 3z + 10 = 0$

[2 marks]

$\alpha =$ ______

3 (c) (ii) Find the values of β and γ

[3 marks]

4 The matrix $\mathbf{M} = \begin{bmatrix} 1 & 4 & 2 \\ 3 & k & 3 \\ 2 & k & 1 \end{bmatrix}$, where k is a constant.

4 (a) Show that $\mathbf{M}$ is non-singular for all values of k.

[3 marks]

4 (b) Obtain $\mathbf{M}^{-1}$ in terms of k.

[5 marks]

4 (c) Use $\mathbf{M}^{-1}$ to solve the equations

$$\begin{aligned} x + 4y + 2z &= 25 \\ 3x + ky + 3z &= 3 \\ 2x + ky + z &= 2 \end{aligned}$$

giving your solution in terms of k.

[4 marks]

$x =$ ______

$y =$ ______

$z =$ ______

5 (a) Show that $\tan x$ is an integrating factor for the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{\sec^2 x}{\tan x}y = \tan x$$

[2 marks]

5 (b) Hence solve this differential equation, given that $y = 3$ when $x = \frac{\pi}{4}$

[6 marks]

Answer

6 Find the eigenvalues and corresponding eigenvectors of the matrix $\mathbf{M} = \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$

[6 marks]

7 (a) Find the values of the constants a, b and c for which $a + bx + cxe^{-3x}$ is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 3x - 8e^{-3x}$$

[5 marks]

$a =$

$b =$

$c =$

7 (b) Hence find the general solution of this differential equation.

[3 marks]

Answer ______________________

7 (c) Hence express y in terms of x, given that $y = 1$ when $x = 0$ and that $\frac{\mathrm{d}y}{\mathrm{d}x} \to -1$ as $x \to \infty$

[4 marks]

Answer ______________________

8 It is given that $y = (4 + \sin x)^{\frac{1}{2}}$

8 (a) Express $y\dfrac{dy}{dx}$ in terms of $\cos x$

[2 marks]

Answer

8 (b) Find the value of $\dfrac{d^3 y}{dx^3}$ when $x = 0$

[5 marks]

Answer

8 (c) Hence, by using Maclaurin's theorem, find the first four terms in the expansion, in ascending powers of x, of $(4 + \sin x)^{\frac{1}{2}}$

[2 marks]

Answer

9 (a) Express $7 + 4x - 2x^2$ in the form $a - b(x - c)^2$, where a, b and c are integers.

[2 marks]

Answer

9 (b) By means of a suitable substitution, or otherwise , find the exact value of

$$\int_{1}^{\frac{5}{2}} \frac{\mathrm{d}x}{\sqrt{7 + 4x - 2x^2}}$$

[6 marks]

Answer

10 The sequence $u_1,\ u_2,\ u_3, \ldots$ is defined by

$$u_1 = \frac{3}{4} \qquad u_{n+1} = \frac{3}{4 - u_n}$$

Prove by induction that, for all $n \geqslant 1$,

$$u_n = \frac{3^{n+1} - 3}{3^{n+1} - 1}$$

[6 marks]

11 (a) Given that

$$x = \ln(\sec t + \tan t) - \sin t$$

show that

$$\frac{dx}{dt} = \sin t \tan t$$

[4 marks]

11 (b) A curve is given parametrically by the equations

$$x = \ln(\sec t + \tan t) - \sin t, \qquad y = \cos t$$

The length of the arc of the curve between the points where $t = 0$ and $t = \dfrac{\pi}{3}$ is denoted by s.

Show that $s = \ln p$, where p is an integer.

[6 marks]

12 A line and plane have equations

$$\frac{x-3}{p} = \frac{y-q}{3} = \frac{z-1}{-1}$$

and

$$\mathbf{r} \cdot \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = 10$$

respectively, where p and q are constants.

12 (a) Show that the line is not perpendicular to the plane.

[1 mark]

12 (b) In the case where the line lies in the plane, find the values of p and q.

[4 marks]

$p =$ ________________

$q =$ ________________

12 (c) In the case where the angle, θ, between the line and the plane satisfies $\sin\theta = \dfrac{1}{\sqrt{6}}$, and the line intersects the plane at $z = 2$:

12 (c) (i) find the value of p

[5 marks]

$p =$ ________________

12 (c) (ii) find the value of q

[2 marks]

$q =$ ________________

13 (a) (i) Express each of the numbers $1 + \sqrt{3}\mathrm{i}$ and $1 - \mathrm{i}$ in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$

[3 marks]

$1 + \sqrt{3}\mathrm{i} =$ ______________________

$1 - \mathrm{i} =$ ______________________

13 (a) (ii) Hence express

$$(1 + \sqrt{3}\mathrm{i})^8 (1 - \mathrm{i})^5$$

in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$

[3 marks]

Answer ______________________

13 (b) Solve the equation

$$z^3 = (1 + \sqrt{3}\mathrm{i})^8 (1 - \mathrm{i})^5$$

giving your answers in the form $a\sqrt{2}\mathrm{e}^{\mathrm{i}\theta}$, where a is a positive integer and $-\pi < \theta \leqslant \pi$

[4 marks]

Answer

END OF QUESTIONS

There are no questions printed on this page

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