OXFORD
INTERNATIONAL
AQA EXAMINATIONS

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS

(9665)

Mark scheme

Further statistics Unit 2

Specimen

Principal Examiners have prepared these mark schemes for specimen papers. These mark schemes have not, therefore, been through the normal process of standardising that would take place for live papers.

Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
ft	Follow through from previous incorrect result
CAO	Correct answer only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
$-x$ **EE**	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

No method shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

O_i	E_i	$\left(\lvert O_i - E_i \rvert - 0.5\right)$ (α)	α^2 / E_i
24	28	3.5	0.4375
56	52	3.5	0.2356
11	7	3.5	1.7500
9	13	3.5	0.9423
			3.3654

Q	Answer	Marks	Total	Comments
1		M1		*E* attempted
		M1		Yates' correction attempted
		M1		χ^2 attempted
	3.3654	A1		AWFW 3.36 to 3.37
	H_0: No association between drug and prevention of sickness	B1		(at least H_0 stated correctly)
	H_1: Association between drug and prevention of sickness			
	$\chi^2_{5\%} = 3.841$	B1		CAO
	Accept H_0	A1ft		
	No evidence at the 5% level of significance to support the claim that the drug is effective against sickness.	E1ft	8	
		Total	**8**	

Q	Answer	Marks	Total	Comments
2 (a)	Sample mean = 380.8	B1		CAO
	$s = 4.38$ or $s^2 = 19.2$	B1		AWRT
	$t_4 = 2.132$	B1		AWRT 2.13
	C.I. = $380.8 \pm 2.132 \times \frac{'4.38'}{\sqrt{5}}$ or $\sqrt{('19.2'/5)}$	M1		Use of their $4.38/\sqrt{5}$ or $\sqrt{('19.2'/5)}$
		m1		Rest of formula (using t_4 or t_5 (2.015))
	= (377, 385)	A1		AWRT
			6	
(b)	3	B1	1	CAO
		Total	**7**	

Q	Answer	Marks	Total	Comments
3	H_0: $\mu_B = \mu_G$ H_1: $\mu_B \neq \mu_G$	B1		At least H_1; allow suffices of 1 & 2 or X & Y, etc
	SL α = 0.05 (5%)			
	CV z = (±)**<u>1.96</u>**	B1		AWRT (1.9600)
	$z = \dfrac{\lvert \bar{b} - \bar{g} \rvert}{\sqrt{\dfrac{\sigma_B^2}{n_B} + \dfrac{\sigma_G^2}{n_G}}} = \dfrac{\lvert 21.35 - 21.90 \rvert}{\sqrt{\dfrac{0.5625}{20} + \dfrac{0.9025}{15}}}$	M1 M1		Numerator Denominator
	= (±)**<u>1.85</u>**	A1		Dependent on at least M1 M0 AWRT (1.8510) Ignore sign (p-value = 0.0642)
	Evidence, at 5% level, that $\boldsymbol{\mu_B = \mu_G}$ **or** **No evidence**, at 5% level, that $\boldsymbol{\mu_B \neq \mu_G}$	A1ft	**6**	ft on CV & z-value; consistent signs Definitive conclusion ⇒ AF0 ft on 5% & p-value; consistent areas
		Total	**6**	

Q	Answer	Marks	Total	Comments
4	95% $\Rightarrow$ z = **1.96**	B1		CAO (AWRT from calculator)
	Require $2 \times \frac{1.96\sigma}{\sqrt{n}} \leq 0.2\mu$	M1		Used; may be implied Allow 'no 2 ×' Allow '= sign' throughout
	Thus $2 \times \frac{1.96}{\sqrt{n}} \times \frac{\mu}{2} \leq 0.2\mu$	M1		Use of $\sigma = \frac{\mu}{2}$; may be implied Allow 'no 2 ×'
	Thus $\sqrt{n} \geq \frac{1.96}{0.2}$	M1		Attempt at solution for $\sqrt{n}$ or n
	Thus $n \geq 96.04$			
	Thus, to nearest 10; n = **100**	A1	5	CAO
		Total	**5**	

Q	Answer	Marks	Total	Comments
5(a)	$H_0 : \sigma^2 = 225 \quad H_1 : \sigma^2 \neq 225$	B1		Both
	$v = 15 - 1 = 14$	B1		
	$\left.\begin{matrix}\chi^2_{14}(0.025) = 5.629 \\ \chi^2_{14}(0.975) = 26.119\end{matrix}\right\}$	B1		Both; or $F(\infty, 14) = 2.487$
	$\chi^2 = \dfrac{(n-1)s^2}{\sigma^2} = \dfrac{14 \times 9.1^2}{225} = 5.15$	M1 A1		$F_{calc} = \dfrac{225}{9.1^2} = 2.72$
	$5.15 < 5.629 \Rightarrow$ Reject H_0			$2.72 > 2.487 \Rightarrow$ Reject H_0
	Evidence to suggest that			
	variance is not 225	A1ft	6	
(b)	$H_0 : \sigma_B^2 = \sigma_G^2 \quad H_1 : \sigma_B^2 \neq \sigma_G^2$	B1		Both
	$\left.\begin{matrix}s_B^2 = 70.567 \\ s_G^2 = 14.25\end{matrix}\right\}$	B1		Both; or $s_B = 8.400 \quad s_G = 3.7749$
	$F_{calc} = \dfrac{70.567}{14.25} = 4.95$	M1 A1ft		AWRT; ft on variances
	$v_1 = 5 \quad v_2 = 3$	B1		
	$F_{5,3} = 14.88$	B1		
	$4.952 < 14.88$			
	$\Rightarrow$ Accept H_0			
	Variances are equal	A1ft	7	
		Total	**13**	

Q	Answer	Marks	Total	Comments
6				
(a)(i)	$E(F) = 128 + 112 =$ **240**	B1	**1**	CAO
(ii)	$\text{Cov}(X, Y) = -0.4 \times \sqrt{50 \times 50} =$ **–20**	M1		Used; or equivalent
	$\text{Var}(F) = 50 + 50 + (2 \times -20) =$ **60**	M1 A1	3	$V(X) + V(Y) + 2\text{Cov}(X,Y)$ used CAO; **AG**
(b)(i)	$E(T) = 240 + 75 =$ **315**	B1ft		ft on **(a)(i)**
	$\text{Var}(T) = 60 + 36 =$ **96**	B1	2	CAO
(ii)	$E(M) = 240 - (3 \times 75) =$ **15**	B1ft		ft on **(a)(i)**
	$\text{Var}(M) = 60 + \{(-3^2) \times 36\}$ $= 60 + 324 =$ **384**	M1 A1	3	$V(F) + 3^2V(S)$ used CAO
(c)	$P(T > 300) = P\left(Z > \frac{300 - 315}{\sqrt{96}}\right)$	M1		Standardising 300 or 300.5 using **(b)(i)**
	$= P(Z > -1.53) = P(Z < 1.53)$	m1		Area change
	= **0.936 to 0.938**	A1	3	AWFW
	Total		**12**	

Q	Answer	Marks	Total	Comments
7(a) (i)	$\mathrm{M}_Z(t) = \mathrm{E}(\mathrm{e}^{tz}) = \int \mathrm{e}^{tz} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} \mathrm{d}z$	M1		Ignore limits
	$= \frac{1}{\sqrt{2\pi}} \int \mathrm{e}^{-\frac{1}{2}(z^2 - 2tz)} \mathrm{d}z = \frac{1}{\sqrt{2\pi}} \int \mathrm{e}^{-\frac{1}{2}(z-t)^2 + \frac{1}{2}t^2} \mathrm{d}z$	m1		Completing square
	$= \mathrm{e}^{\frac{1}{2}t^2} \times \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{e}^{-\frac{1}{2}u^2} \mathrm{d}u$ where $u = z - t$	m1		Removing constant term and substitution
	$= \mathrm{e}^{\frac{1}{2}t^2} \times 1 = \mathrm{e}^{\frac{1}{2}t^2}$	A1	4	Fully correct derivation
(ii)	$\mathrm{M}_X(t) = \mathrm{M}_{\mu + \sigma z}(t) = e^{\mu t} \mathrm{M}_Z(\sigma t)$	M1		Substitution and two terms
	$= \mathrm{e}^{\mu t} \times \mathrm{e}^{\frac{1}{2}(\sigma t)^2} = \mathrm{e}^{\mu t + \frac{1}{2}\sigma^2 t^2}$	A1	2	Fully correct deduction
(iii)	Mean $= \mathrm{M}'_X(0) = \left[\left(\mu + \sigma^2 t\right)\mathrm{e}^{\mu t + \frac{1}{2}\sigma^2 t^2}\right]_{t=0} = \mu$	M1 A1		Attempt at first derivative Correct evaluation at $t = 0$
	$\mathrm{M}''_X(0) = \left[\left(\sigma^2\right)\mathrm{e}^{\mu t + \frac{1}{2}\sigma^2 t^2} + \left(\mu + \sigma^2 t\right)^2 \mathrm{e}^{\mu t + \frac{1}{2}\sigma^2 t^2}\right]_{t=0}$	M1		Attempt at second derivative
	$= \sigma^2 + \mu^2$	A1		Correct evaluation at $t = 0$
	Variance $= \mathrm{M}''_X(0) - (\text{mean})^2$ $= \sigma^2 + \mu^2 - \mu^2 = \sigma^2$	B1	5	CAO
(b) (i)	$\mathrm{M}_{\bar{X}}(t) = M_{\sum X_i}\left(\frac{t}{n}\right)$	B1		Catering for $1/n$
	Using product of mgf's	M1		

	$= \left(e^{\mu \frac{t}{n} + \frac{1}{2}\sigma^2 \left(\frac{t}{n}\right)^2} \right)^n$	m1		Attempted application
	$= e^{\mu t + \frac{\sigma^2}{2n} t^2}$	A1	4	Fully correct deduction
(ii)	$\bar{X}$ has normal distribution with mean μ	Bdep1		Normal and μ; dependent on (b)(i)
	and variance σ^2/n	Bdep1	2	Dependent on (b)(i)
		Total	**17**	

Q	Answer	Mark	Total	Comment
8(a)	$\mathrm{E}(\bar{X}_1) = \frac{n_1\mu}{n_1} = \mu$ and $\mathrm{E}(\bar{X}_2) = \frac{n_2\mu}{n_2} = \mu$ $\mathrm{E}(k\bar{X}_1 + (1-k)\bar{X}_2) = k\mathrm{E}(\bar{X}_1) + (1-k)\mathrm{E}(\bar{X}_2)$ $= k\mu + (1-k)\mu = \mu$	M1 A1	2	Stated or implied.
(b)	$\mathrm{Var}(k\bar{X}_1 + (1-k)\bar{X}_2)$ $= k^2\mathrm{Var}(\bar{X}_1) + (1-k)^2\mathrm{Var}(\bar{X}_2)$ $\mathrm{Var}(\bar{X}_1) = \frac{\sigma^2}{n_1}$ and $\mathrm{Var}(\bar{X}_2) = \frac{\sigma^2}{n_2}$ (OE) $\Rightarrow V = k^2\frac{\sigma^2}{n_1} + (1-k)^2\frac{\sigma^2}{n_2}$ (AG)	M1 A1	2	Stated or implied.
(c)	$\frac{\mathrm{d}V}{\mathrm{d}k} = \sigma^2\left\{\frac{2k}{n_1} - \frac{2(1-k)}{n_2}\right\}$ $\frac{k}{n_1} - \frac{(1-k)}{n_2} = 0 \Rightarrow k = \frac{n_1}{n_1+n_2}$	M1A1 A1	3	Using $n_1 = n_2 = n$ from the start ⇒M0.
(d)(i)	$k\bar{X}_1 + (1-k)\bar{X}_2 = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1+n_2}$ (OE)	M1A1ft	2	F.t. on algebraic form. $\frac{1}{2}$ gets A0.
(ii)	This is the weighted average of means.	E1	1	Explanation in terms of proportionality, or 'pooled estimate' OK.
(iii)	$\frac{\mathrm{d}^2V}{\mathrm{d}k^2} = 2k\sigma^2\left\{\frac{1}{n_1} + \frac{1}{n_2}\right\} > 0 \Rightarrow$ minimum V.	M1A1	2	No omissions.
	Total		**12**	

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