OXFORD
INTERNATIONAL AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS

(FM04) Unit FS2 – Further Statistics

Specimen 2018   Morning   Time allowed: 1 hour 30 minutes

Materials

- For this paper you must have the booklet of formulae and statistical tables.
- You may use a graphics calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use a supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box or around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions in the spaces provided.

1 It is claimed that a new drug is effective in the prevention of sickness in tourists.
A sample of 100 tourists was surveyed with the following results.

	Sickness	**No Sickness**	**Total**
Drug taken	24	56	80
No drug taken	11	9	20
Total	35	65	100

Assuming that the 100 tourists are a random sample, use a χ^2 test, at the 5% level of significance, to investigate the claim.

[8 marks]

2 Vanya collected five samples of air and measured the carbon dioxide content of each sample, in parts per million by volume (ppmv). The results were

387	375	382	379	381

(a) Assuming that these data form a random sample from a normal distribution with mean μ ppmv, construct a 90% confidence interval for μ.

[6 marks]

Answer ______________________________

(b) Vanya repeated her sampling procedure on each of 30 days and, for each day's results, a 90% confidence interval for μ was constructed.

On how many of these 30 days would you expect μ to lie outside that day's confidence interval?

[1 mark]

Answer ______________________________ days

3 Each household within an area has two types of bin: a black one for general trash and a green one for garden trash.

The mass, in kilograms, of trash emptied from a black bin can be modelled by the random variable $B \sim \mathrm{N}(\mu_B, 0.5625)$.

The mass, in kilograms, of trash emptied from a green bin can be modelled by the random variable $G \sim \mathrm{N}(\mu_G, 0.9025)$.

The mean mass of trash emptied from a random sample of 20 black bins was 21.35 kg. The mean mass of trash emptied from an independent random sample of 15 green bins was 21.90 kg.

Test, at the 5% level of significance, the hypothesis that $\mu_B = \mu_G$

[6 marks]

4 The waiting time at a hospital's Accident and Emergency department may be approximated by a normal distribution with mean μ and standard deviation $\frac{\mu}{2}$.

The department's manager wishes a 95% confidence interval for μ to be constructed such that it has a width of at most 0.2μ.

Calculate, to the nearest 10, an estimate of the minimum sample size necessary in order to achieve the manager's wishes.

[5 marks]

5 (a) The IQs of a random sample of 15 students have a standard deviation of 9.1

Test, at the 5% level of significance, whether this sample may be regarded as coming from a population with a variance of 225. Assume that the population is normally distributed.

[6 marks]

(b) The masses, in kilograms, of 6 boys and 4 girls were found to be as follows.

Boys	53	37	41	50	57	57
Girls	40	46	37	40		

Assume that these data are independent random samples from normal populations.

Show that, at the 5% level of significance, the hypothesis that the population variances are equal is accepted.

[7 marks]

6 An aircraft, based at airport A, flies regularly to and from airport B.

The aircraft's flying time, X minutes, from A to B has a mean of 128 and a variance of 50.

The aircraft's flying time, Y minutes, on the return flight from B to A is such that

$$\text{E}(Y) = 112, \quad \text{Var}(Y) = 50 \quad \text{and} \quad \rho_{xy} = -0.4$$

(a) Given that $F = X + Y$:

(i) find the mean of F;

[1 mark]

Answer ______________________

(ii) show that the variance of F is 60

[3 marks]

(b) At airport B, the stopover time, S minutes, is independent of F and has a mean of 75 and variance of 36.

Find values for the mean and variance of:

(i) $T = F + S$;

[2 marks]

mean = ____________________

variance = ____________________

(ii) $M = F - 3S$.

[3 marks]

mean = ____________________

variance = ____________________

(c) Hence, assuming that T is normally distributed, determine the probability that, on a particular round trip of the aircraft from A to B and back to A, the time from leaving A to returning to A exceeds 300 minutes.

[3 marks]

Answer ____________________

7 The random variable Z has the probability density function

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} \quad -\infty < z < \infty$$

(a)(i) Derive the moment generating function, $M_Z(t)$, of Z.

[4 marks]

(ii) Deduce the moment generating function, $M_X(t)$, of $X = \mu + \sigma Z$.

[2 marks]

(iii) Hence find the mean and the variance of X.

[5 marks]

mean = ________________

variance = ________________

(b) The random variable $\overline{X}$ is defined by $\overline{X} = \frac{1}{n}\sum X_i$, where X_i are independently distributed, each with moment generating function $M_{\overline{X}}(t)$.

(i) Determine an expression for $M_{\overline{X}}(t)$

[4 marks]

Answer

(ii) Hence specify completely the distribution of $\overline{X}$

[2 marks]

8 Two independent random samples of observations, of sizes n_1 and n_2, are made of random variable X, which has mean μ and variance σ^2. The sample means are denoted by $\overline{X}_1$ and $\overline{X}_2$ respectively.

(a) Show that $T = k\overline{X}_1 + (1 - k)\,\overline{X}_2$ is an unbiased estimator of μ.

[2 marks]

(b) Show that V, the variance of T, is given by

$$V = k^2\frac{\sigma^2}{n_1} + (1 - k)^2\frac{\sigma^2}{n_2}$$

[2 marks]

(c) Find the value of k for which $\frac{dV}{dk} = 0$

[3 marks]

Answer

(d) For the value of k found in part **(c)**:

(i) find an expression for T;

[2 marks]

Answer

(ii) interpret the expression found in part **(d)(i)**;

[1 mark]

(iii) Find $\frac{d^2V}{dk^2}$ and hence comment on what you can deduce about V.

[2 marks]

Answer

END OF QUESTIONS