OXFORD
INTERNATIONAL AQA EXAMINATIONS

INTERNATIONAL A-LEVEL FURTHER MATHEMATICS

(9665)

Mark scheme

Further mechanics Unit 2

Specimen

Principal Examiners have prepared these mark schemes for specimen papers. These mark schemes have not, therefore, been through the normal process of standardising that would take place for live papers.

Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
ft	Follow through from previous incorrect result
CAO	Correct and answer only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
$-x$ **EE**	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

No method shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Answer	Marks	Total	Comments
1(a)	$\text{EPE} = \frac{\lambda x^2}{2l}$			
	$= \frac{180 \times 0.8^2}{2 \times 1.2}$	M1		
	$= 48$ J	A1	2	
(b)	Using initial EPE = KE when string becomes slack:	M1		
	$48 = \frac{1}{2} \times 5 \times v^2$	A1ft		
	$v = \sqrt{\frac{96}{5}}$			
	$= 4.38$ m s^{-1}	A1ft	3	ft $\sqrt{\frac{'a'}{2.5}}$
(c)	Normal reaction is $5g$ or 49	M1		
	Frictional force is $5g \times \mu$	m1A1		
	Work done by frictional force is $5\mu g \times 2$	m1		
	$= 10\mu g$	A1		
	Stops at wall $\Rightarrow$ $10\mu g = 48$	m1		m1 $10\mu g = 'a'$
	$\mu = 0.490$	A1	7	accept $\frac{24}{49}$ OE
	Total		**12**	

Q	Answer	Marks	Total	Comments
2(a)	Using conservation of energy:			
	$\frac{1}{2}m(5u)^2 = \frac{1}{2}m(2u)^2 + 2amg$	M1A1		M1 for 3 [or 4] terms: 2 KE and 1[or 2] PE
	$\frac{1}{2} \times 21 \times u^2 = 2ag$	M1		M1A1 for finding h
	$u = \sqrt{\frac{4ag}{21}}$	A1	4	
(b)	Using conservation of energy with speed at point S to be V:			**Or**
	$\frac{1}{2}m(5u)^2 = \frac{1}{2}m(V)^2 + amg(1+\cos 60)$	M1		$\frac{1}{2}m(V)^2 = amg(1-\cos 60°) + \frac{1}{2}m\left(2\sqrt{\frac{4ag}{21}}\right)^2$
	$\frac{1}{2}mV^2 = \frac{1}{2}m(5u)^2 - 1\frac{1}{2}amg$			
	$V^2 = 25 \times \left(\frac{4ag}{21}\right) - 3ag$			
	$V^2 = \frac{37ag}{21}$	A1		
	Resolving radially at point S:			
	$R = -mg\cos 60 + \frac{m(V)^2}{a}$	M1A1		
	$= -\frac{1}{2}mg + \frac{37mg}{21}$			
	$= \frac{53}{42}mg$ or $1.26mg$	A1	5	
	Total		**9**	

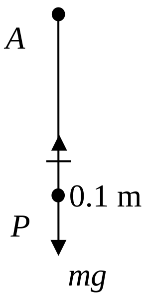
Q	Answer	Marks	Total	Comments
3 (a)	$0.2\frac{dv}{dt} = -0.2g - 0.01v^2$	M1		
	$\frac{dv}{dt} = -g - \frac{v^2}{20}$			
	$= -\left(\frac{20g+v^2}{20}\right)$	A1	2	
(b)	$\int \frac{20}{20g+v^2}\, dv = -\int dt$ B1	B1		
	$\frac{20}{\sqrt{20g}} \tan^{-1} \frac{v}{\sqrt{20g}} = -t + c$ M1 A1	M1 A1		(condone omission of c)
	$t = 0,\ v = 10$			
	hence $c = \frac{20}{\sqrt{20g}} \tan^{-1} \frac{10}{\sqrt{20g}}$ M1	M1		
	when $= 0$, $t = \frac{20}{\sqrt{20g}}$ $\tan^{-1} \frac{10}{\sqrt{20g}}$	A1		(In part **(b)** there is a straightforward alternative using definite integration)
	$t = 0.89$	A1	6	
	Total		**8**	

Q	Answer	Marks	Total	Comments
4 (a)	Perpendicular to the plane:			
	$y = -\frac{1}{2}gt^2\cos 20 + ut\sin 30$	M1		
	$0 = -4.9t^2\cos 20 + ut\sin 30$	M1		
	$t = 0.108589568u$ or $\frac{2u\sin 30}{g\cos 20}$	A1		
	Parallel to the plane:			
	$x = -\frac{1}{2}gt^2\sin 20 + ut\cos 30$	M1		
	$200 = -4.9(0.108589568u)^2\sin 20 +$ $u(0.108589568u)\cos 30$	m1		
	$u^2 = 2693$	A1ft		
	$u = 51.9$ or 51.894	A1ft	7	Do not accept $\sqrt{2693}$
(b)	$\dot{y} = -gt\cos 20 + u\sin 30 = 0$	M1		
	$t = 2.817899$ or 2.817580214 or $\frac{51.9\sin 30}{g\cos 20}$	A1ft		Accept 3 significant fig.
	The greatest $\perp$ distance = $-\frac{1}{2}9.8(2.817899)^2\cos 20 + 51.9(2.817899)\sin 30$ or	m1		
	$-\frac{1}{2}9.8(\frac{51.894\sin 30}{9.8\cos 20})^2\cos 20 + 51.9(\frac{51.894\sin 30}{9.8\cos 20})\sin 30$			
	$= 36.5622$ m or 36.5538 $= 36.6$ 3sf	A1ft	4	
	Total		**11**	

4 (a)	**Alternative:**			
	$x = 200\cos 20$	B1		
	$y = 200\sin 30$	B1		
	$200\cos 20 = u\cos 50t$	M1		
	$t = \dfrac{292.4}{u}$	A1		
	$200\sin 30 = \dfrac{1}{2}(-9.8)(\dfrac{292.4}{u})^2 + u\sin 50(\dfrac{292.4}{u})$	M1		
	$u^2 = 2693$	A1		
	$u = 51.9$	A1		
(b)	**Alternative:**			
	$0 = (u\sin 30)^2 - 2g\cos 20.s$	M1		
	$s = \dfrac{(51.9\sin 30)^2}{2(9.8)\cos 20}$	m1A1		
	$s = 36.6$	A1		

Q	Answer	Marks	Total	Comments
5(a)	C.L.M. $(1)3u = (1)v_A + (3)v_B$	M1 A1		M1 for three non-zero terms
	Restitution : $\frac{1}{3} \times 3u = v_B - v_A$	M1 A1		Accept $v_A - v_B$
	$v_B = u$	m1		Solution
	$v_A = 0$	A1	6	A1 for both answers
(b)	C.L.M. $3u = 3w_B + xw_c$	M1 A1		
	Restitution : $\frac{1}{3}u = w_C - w_B$	M1 A1		
	$w_C = \frac{4u}{3+x}$	m1		Solution attempt, dep. on both M1s AG
	$w_B = \frac{u(9-x)}{3(3+x)}$ OE	A1	6	A1 for both
(c)	For further collision $\frac{u(9-x)}{3(3+x)} < 0$	M1		
	$9u - xu < 0$			
	$x > 9$	A1	2	AG
(d)	$I = 5(\frac{4u}{3+5})$	M1		
	$I = \frac{5u}{2}$	A1	2	
	Alternative: $I = 3u - 3 \times \frac{u(9-5)}{3(3+5)}$	(M1)		
	$I = \frac{5u}{2}$	(A1ft)		Accept $-\frac{5u}{2}$ Follow through on their w_B
	Total		**16**	

Q	Answer	Marks	Total	Comments
6 (a)	(Let vB = ai – bj)			
	$\frac{a}{b} = \frac{3}{2}$	M1		
	$\frac{a}{b} = \frac{3}{2}$	A1		Allow sign error
	(Squares are smooth ⇒ j component unchanged) $b = 3$	B1		OE
	$a = \frac{9}{2}$	A1	4	AG
	$\left(v_B = \frac{9}{2}\mathbf{i} - 3\mathbf{j}\right)$			
(b)	(C.L.M. along the line of centres:)			
	$4(4) - 2(2) = 4(v_A) + 2(\frac{9}{2})$	M1		
	$v_A = \frac{3}{4}$	A1		OE, No sign errors
	(Restitution along the line of centres:)			
	$e = \frac{-\frac{3}{4} + \frac{9}{2}}{4 + 2}$ OE	M1 A1		M1 for correct terms, A0 for sign error
	$e = \frac{5}{8}$	A1	5	
(c)	(I = Change in momentum of B along the line of centres)			
	$= 2\,(\frac{9}{2}\mathbf{i}) - 2\,(-2\mathbf{i})$	M1		Allow sign error and missing i
	$= 13\mathbf{i}$	A1		A0 for magnitude or –13i
	Ns or kg m s^{-1}	B1	3	
	Total		**12**	

Q	Answer	Marks	Total	Comments
7(a)	$T = 0.4g$			
				
	$T = 0.1k$	M1		Both, accept use of λ and l
	$0.1k = 0.4g$	A1		
	$k = 4g$			
	$k = 39.2\ \text{N}\,\text{m}^{-1}$	A1ft	3	
(b)	extension = $0.1 + x$	B1		
(i)	$mg - T = m\ddot{x}$	M1		
	$0.4g - 39.2(x + 0.1) = 0.4\ddot{x}$	A1ft		ft stiffness
	$0.4\ddot{x} = -39.2x$			
	$\ddot{x} = -98x$ $(h = -98)$	A1ft	4	ft stiffness
(ii)	$\ddot{x} = -\omega^2 x$ SHM (constant < 0)	B1ft	1	ft stiffness provided $h < 0$
(iii)	$T = \frac{2\pi}{\omega}$			
	$T = \frac{2\pi}{\sqrt{98}}$	M1		
	$T = 0.63$ s	A1	2	AG
(iv)	max $v = a\omega$			
	$= 0.2 \times \sqrt{98}$	M1		
	$= 2.0\ \text{m}\,\text{s}^{-1}$	A1ft	2	ft stiffness provided $h < 0$
	Total		**12**	

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