OXFORD
INTERNATIONAL
AQA EXAMINATIONS

INTERNATIONAL AS MATHEMATICS

(9660)

MA01: Pure Mathematics Unit 1

Example responses with commentary

For teaching from September 2016 onwards
For AS exams in May/June 2018 onwards

This guide includes questions from the summer 2018 MA01: Pure Mathematics Unit 1 exam.

It is intended to provide examples of genuine responses with some exemplification of the mark scheme connected to that response. Each series, the intention is to provide further example responses as they become available.

QUESTION

01 (b)

1 (b) The graph of $y = x^2$ is translated onto the graph of $y = x^2 + 6x + 14$ by a vector.

Find this vector.

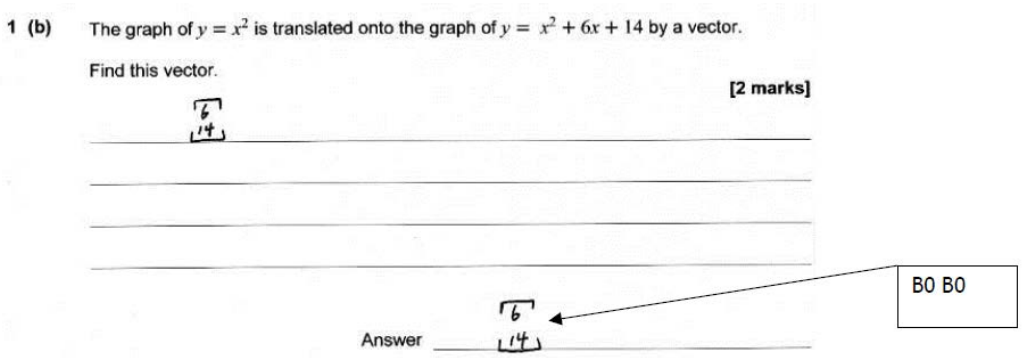
[2 marks]

Answer _____

MARK SCHEME

Q	Answer	Mark	Comments
1(b)	$\begin{pmatrix} -3 \\ 5 \end{pmatrix}$	B1ft B1ft	B1 for each component ft minus their a ft their b

STUDENT A


1 (b) The graph of $y = x^2$ is translated onto the graph of $y = x^2 + 6x + 14$ by a vector.

Find this vector.

[2 marks]

$\begin{bmatrix} 6 \\ 14 \end{bmatrix}$

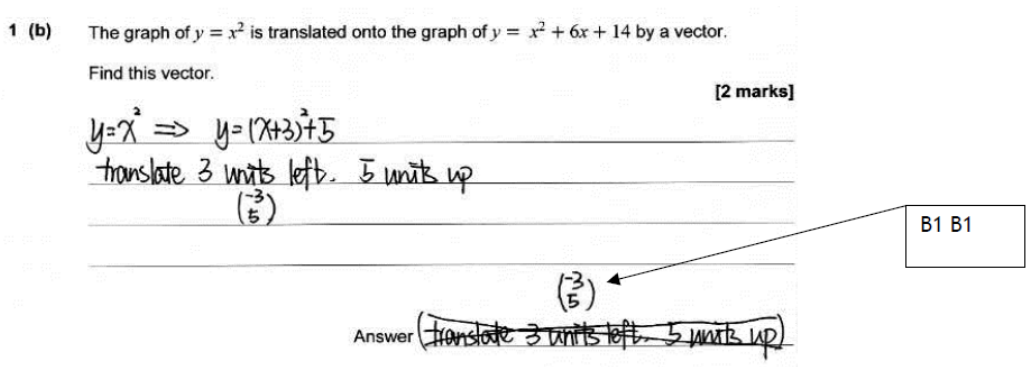
Answer $\begin{bmatrix} 6 \\ 14 \end{bmatrix}$

B0 B0

EXAMINER COMMENTARY

An example of a common but totally incorrect answer to this question. Part (a) required the student to write the equation of the graph after the translation in completed square form thus giving the required elements to enter into the vector in part (b). If the student had incorrectly answered part (a) and then gone on to use their values correctly in this part then they would have gained both marks.
Unfortunately, however, this is also incorrect vector notation and would have gained no marks even if the correct values were given.
In this question it was also common to see students giving the correct values but as coordinates. These students also scored no marks for incorrect notation.

STUDENT B



EXAMINER COMMENTARY

An example of a fully correct answer with one mark for each correct value given using the correct notation. If, however, the student had not crossed out their written description of the translation on the answer line then no marks would have been awarded. The question asked for the vector and not a written description of the translation and if not crossed out this would have presented a choice of answer.
Also, if the answer given on the answer line was solely the written description no marks would have been given even though the correct answer can be seen in the working space.

QUESTION

02 (b)

2 (b) Show that $\frac{3\sqrt{7}-4\sqrt{6}}{2\sqrt{7}+\sqrt{6}}$ can be written in the form $\frac{p-\sqrt{q}}{r}$ where p, q and r are integers.

Fully justify your answer.

[4 marks]

MARK SCHEME

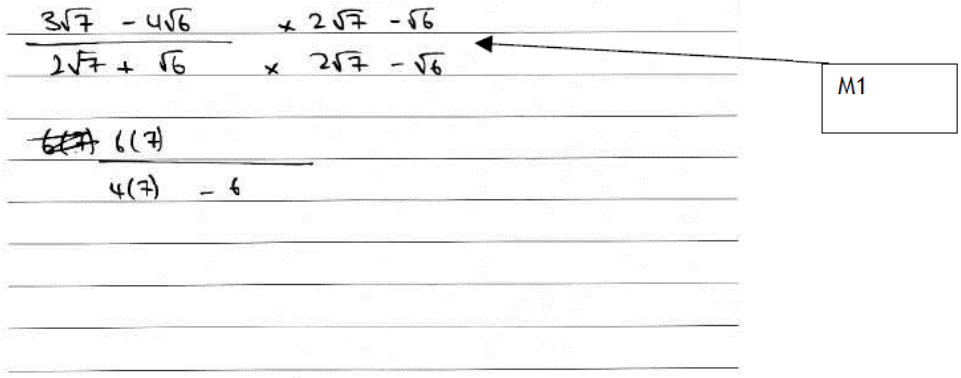
Q	Answer	Mark	Comments
2(b)	$\frac{3\sqrt{7}-4\sqrt{6}}{2\sqrt{7}+\sqrt{6}} \times \frac{2\sqrt{7}-\sqrt{6}}{2\sqrt{7}-\sqrt{6}}$	M1	Multiplies numerator and denominator by the conjugate of the denominator
	(Numerator =) $6(\sqrt{7})^2 - 3\sqrt{7}\sqrt{6} - 8\sqrt{6}\sqrt{7} + 4(\sqrt{6})^2$ or $42 - 3\sqrt{42} - 8\sqrt{42} + 24$	M1	Must be a correct four or three term expression. Allow one error
	(Denominator =) $(2\sqrt{7})^2 - (\sqrt{6})^2$ or $28 - 6$ or 22	B1	Must be seen as a denominator
	$\frac{6-\sqrt{42}}{2}$	A1	NMS = 0

STUDENT A

2 (b) Show that $\frac{3\sqrt{7}-4\sqrt{6}}{2\sqrt{7}+\sqrt{6}}$ can be written in the form $\frac{p-\sqrt{q}}{r}$ where p, q and r are integers.

Fully justify your answer.

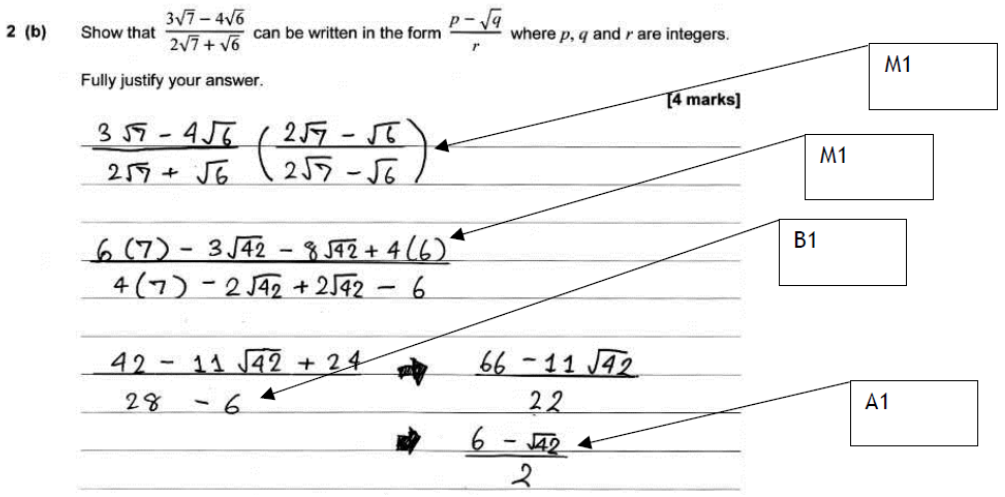
[4 marks]



EXAMINER COMMENTARY

The first method mark in this question was for showing that numerator and denominator are both to be multiplied by the conjugate of the denominator. This is an example of the minimum acceptable for this mark to be awarded as it is clear that it is this student's intention to multiply **both** numerator and denominator, even though the conjugates are not in brackets. Omission of the multiplication signs here would have resulted in this mark not being awarded.

STUDENT B



EXAMINER COMMENTARY

This is an example of a fully correct answer. At each stage in the process the student is clearly demonstrating their technique.
This question required the student to fully justify their answer. Consequently, if they had not shown a three or four-term expression for the expanded numerator but had just written the fully simplified expression immediately then the second method mark could not have been awarded and consequently the final accuracy mark would not have been given, even though they have obtained the correct final solution. For full marks in this question the student needed to show multiplication of numerator and denominator by the conjugate, a correct three or four-term expression for the expanded numerator (one error allowed), the correct denominator shown as a denominator (simplified or not simplified) and finally a correct fully simplified final answer in the required form.
Note, students are expected to give a fully simplified final answer. Failure to have done this in this case would have resulted in the final accuracy mark not being awarded.

QUESTION

03 (a) (ii)

3 (a) (ii) Find the coordinates of the point where L intersects the x-axis.

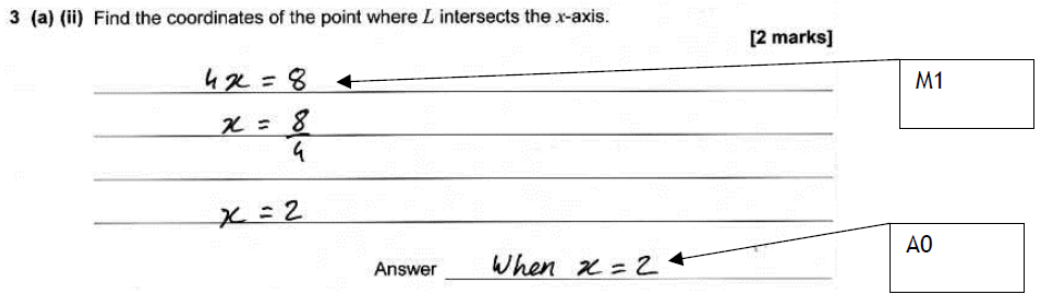
[2 marks]

Answer _____

MARK SCHEME

Q	Answer	Mark	Comments
3(a)(ii)	$4x - 5 \times 0 = 8$	M1	Substituting $y = 0$ into equation of L
	$(2, 0)$	A1	

STUDENT A


3 (a) (ii) Find the coordinates of the point where L intersects the x-axis.

[2 marks]

$4x = 8$

$x = \frac{8}{4}$

$x = 2$

Answer When $x = 2$

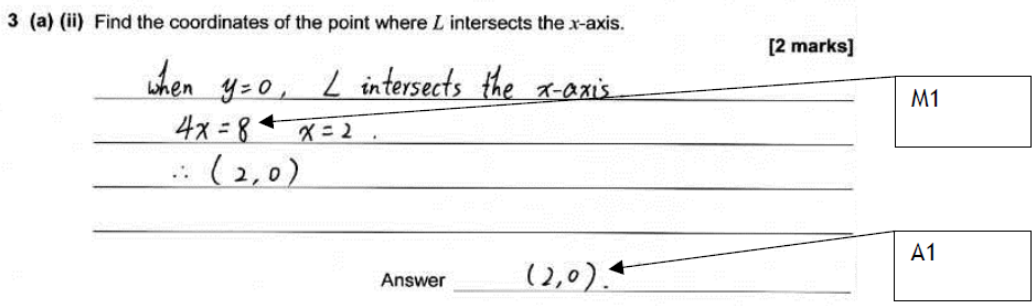
M1

A0

EXAMINER COMMENTARY

This was a common error in this question. The student obtained the method mark for substituting $y = 0$ into the equation for *L* given earlier, however only the x-coordinate is given for their final answer, so the accuracy mark is not given as the question specifically asks for the coordinates of the point of intersection.

STUDENT B


3 (a) (ii) Find the coordinates of the point where L intersects the x-axis.

[2 marks]

when $y=0$, L intersects the x-axis

$4x = 8$ $x = 2$.

$\therefore (2, 0)$

Answer $(2,0)$.

M1

A1

EXAMINER COMMENTARY

A totally correct solution with the method mark being award for the correct substitution of $y = 0$ and the accuracy mark given for the correct coordinates. If, however, the student had given '$x = 2$ and $y = 0$' as their final answer then the accuracy mark would not have been awarded as their answer would not have been given as coordinates.
The correct coordinates given with no working would have scored both marks.

QUESTION

03 (b) (ii)

3 (b) (ii) Find the equation of the line that passes through A and B.

Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

[3 marks]

Answer _____

MARK SCHEME

Q	Answer	Mark	Comments
3(b)(ii)	their gradient of $AB \times (x-4)$ $= y-9$	M1	or y = (their m)$x + c$ and attempt at c using $x = 4, y = 9$ or $x = 2, y =$ their k
	$\frac{-5}{4}(x-4) = y-9$ or $y = 14 - \frac{5}{4}x$	A1	
	$5x + 4y - 56 = 0$ or $-5x - 4y + 56 = 0$	B1	

STUDENT A

3 (b) (ii) Find the equation of the line that passes through A and B.

Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

[3 marks]

M1A1

$y - 9 = -\frac{5}{4}(x - 4)$

$5x + 4y - 56$

$y - 9 = -\frac{5x}{4} + 5$

$\frac{5x}{4} + y - 14 = 0$

Answer $5x + 4y - 56$

B0

EXAMINER COMMENTARY

In this part of the question the student was required to find the equation of a line perpendicular to another line, the gradient of which they had found earlier in the question.

The method and accuracy marks were awarded for correctly using the correct gradient they had found earlier to form a correct equation for the required line in any form. If the gradient they had found earlier had been incorrect, but they had correctly used it to find a value for the gradient of the line through *A* and *B*, then they would have scored M1A0 for forming an equation for their line.

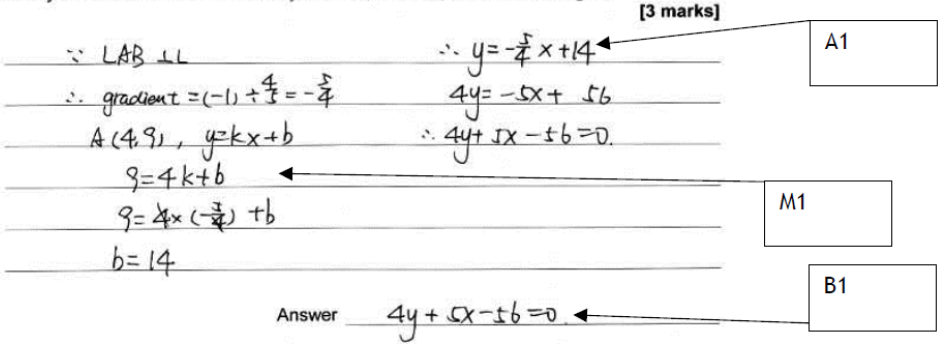
Their final answer is not set to equal zero and therefore is not correct and does not score the final mark.

STUDENT B

3 (b) (ii) Find the equation of the line that passes through A and B.

Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

[3 marks]



EXAMINER COMMENTARY

In this fully correct alternative method the student has correctly used the given coordinates of *A* and the gradient found earlier in an attempt to find c in their $y = mx + c$, thus gaining the method mark. The accuracy mark was awarded for finding the correct equation in any form.

The final mark was awarded for stating the correct equation in the form required in the question even though they wrote the x and y terms in the opposite order (this mark was for a correct equation set equal to zero with integer coefficients). Any correct multiple would have gained the final mark as long as it was of the correct form with integer values.

Common mistakes leading to the loss of this final mark were arithmetic errors rearranging the equation or leaving the coefficients as fractions.

QUESTION

04 (a) (ii)

4 (a) The nth term of a sequence is u_n

This sequence is defined by

$$u_{n+1} = ku_n + 17$$

where k is a constant.

The first two terms of this sequence are $u_1 = 5$ and $u_2 = 23$

4 (a) (ii) Find the values of u_3 and u_4

[2 marks]

$u_3 =$ _____ $u_4 =$ _____

MARK SCHEME

Q	Answer	Mark	Comments
4(a)(ii)	$u_3 = 44.6$	B1	oe
	$u_4 = 70.52$	B1ft	oe. ft their $u_3 \times 1.2 + 17$

STUDENT A

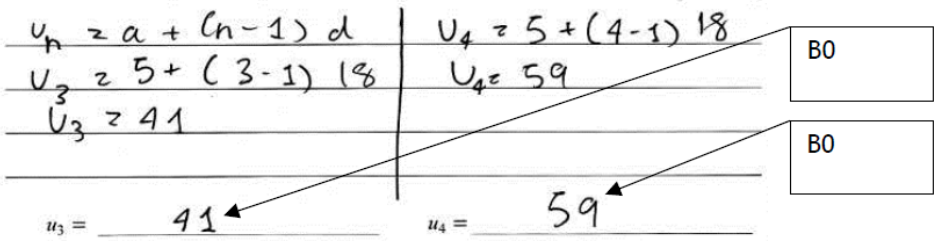
4 (a) The nth term of a sequence is u_n

This sequence is defined by $u_{n+1} = ku_n + 17$

where k is a constant.

The first two terms of this sequence are $u_1 = 5$ and $u_2 = 23$

4 (a) (ii) Find the values of u_3 and u_4 **[2 marks]**



EXAMINER COMMENTARY

This is an example of a totally incorrect answer. The previous part was to find the value of k. In this part the value of k was to be used to find the third and fourth terms of the sequence with a mark given for each one correct.
A common misunderstanding was to assume that it was a geometric sequence with common ratio k, or in this case an arithmetic sequence. It was evident that some students were misinterpreting some questions.
In this case if the student had gone on from 41 as their third term and used it in the given definition with the correct value of k to evaluate the correct value for the fourth term corresponding to 41 then they would have gained the second mark only.

STUDENT B

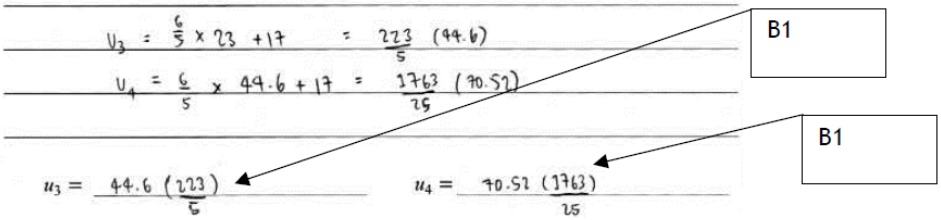
4 (a) The nth term of a sequence is u_n

This sequence is defined by $u_{n+1} = ku_n + 17$

where k is a constant.

The first two terms of this sequence are $u_1 = 5$ and $u_2 = 23$

4 (a) (ii) Find the values of u_3 and u_4 **[2 marks]**



EXAMINER COMMENTARY

A correct answer with one mark for each correct value that can either be given as a fraction or an exact decimal.
Some students, however, showed no working and gave the value of the fourth term as 70.5 thus not gaining the second mark. Students should not round in these instances, and if rounding is appropriate the required accuracy will normally be stated in the question.

QUESTION

05 (c)

5 (c) The curve C has equation $y = (1 + 2x)^3$ and the line L has equation $y = 3 - 10x$

Show that there is **no** point on C at which the tangent to C is parallel to L.

[4 marks]

MARK SCHEME

Q	Answer	Mark	Comments
5(c)	$6 + 24x + 24x^2 = -10$	M1	ft expression for $\frac{dy}{dx}$ from Question **5(b)**
	$3x^2 + 3x + 2 = 0$ or $24x^2 + 24x + 6 + 10 = 0$ or $24x^2 + 24x + 16 = 0$	M1	Rearranges their quadratic equation (simplified or unsimplified) with RHS equal to zero
	Discriminant = -15	B1ft	Correctly calculates the discriminant for their quadratic equation
	Since the discriminant is negative there are no real roots and therefore C does not have a tangent parallel to L	E1	Correct conclusion from totally correct working Be convinced

STUDENT A

5 (c) The curve C has equation $y = (1 + 2x)^3$ and the line L has equation $y = 3 - 10x$

Show that there is **no** point on C at which the tangent to C is parallel to L. **[4 marks]**

gradient of $L = -10$.

if there is a point on C tangent to C is parallel to L

$\therefore 6 + 24x + 24x^2 = -10$

$24x^2 + 24x + 16 = 0$

$\Delta = 576 - 4 \times 24 \times 16 = -960$

$\therefore$ no solution

$\therefore$ there is no point on C tangent to L is parallel to L

M1

M1

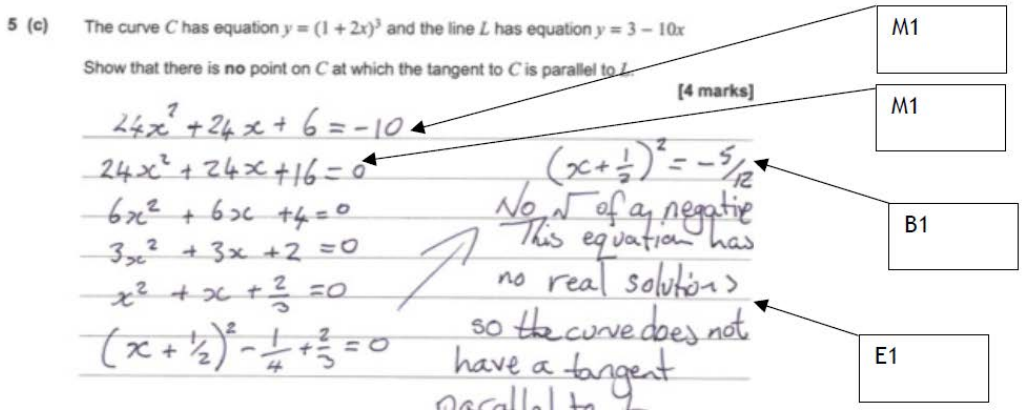
B1

E0

EXAMINER COMMENTARY

In this example the student has correctly set the derivative of the equation for the curve *C* (found earlier in the question) to -10 and so earnt the first method mark. The second method mark was awarded for rearranging the equation correctly with zero on the right-hand side. The third mark was awarded for correctly calculating the value of the discriminant.
However, the conclusion lacked the required detail. If the student had indicated that the discriminant was negative before stating there were no solutions then the explanation would have been complete and would then have been awarded the final mark for the explanation and conclusion.
It is important that a student completely justifies any conclusions.

STUDENT B



EXAMINER COMMENTARY

In this example of an alternative method the student has correctly set the derivative of the equation for the curve *C* to -10 and so earnt the first method mark. The second method mark was awarded for rearranging the equation correctly with zero on the right-hand side. The third mark was given for arriving at a correct rearrangement of the quadratic with a correct bracketed and squared linear term on one side and a negative constant on the other. The final mark has been awarded in this case since the conclusion is stated after a complete, logical and correct argument.

QUESTION

07 (b) (i)

7 The terms of a sequence with first term u_1 are given by

$$u_1 = 672$$

$$u_{n+1} = -\frac{3}{4}u_n$$

7 (b) (i) State how you know that the series $u_1 + u_2 + u_3 + \ldots$ converges.

[1 mark]

MARK SCHEME

Q	Answer	Mark	Comments
7(b)(i)	$r = \frac{-3}{4}$ and $\lvert r \rvert < 1$ or $\left\lvert \frac{-3}{4} \right\rvert < 1$ or $-1 < r < 1$ or $-1 < \frac{-3}{4} < 1$	E1	Must state the correct value for r and give the condition for convergence

STUDENT A

7 The terms of a sequence with first term u_1 are given by

$$u_1 = 672$$

$$u_{n+1} = -\frac{3}{4}u_n$$

7 (b) (i) State how you know that the series $u_1 + u_2 + u_3 + \ldots$ converges. **[1 mark]**

$\frac{672}{1+\frac{3}{4}} = 384$ Because its $|r| = |-\frac{3}{4}| = \frac{3}{4} < 1$

E1

EXAMINER COMMENTARY

In this example the student has given both the correct value of the common ratio and stated that its absolute value is less than one. This response merits the mark.

The student has also correctly calculated the sum to infinity of the series. In this case it has been ignored and the mark given. However, some students only found the sum to infinity, assuming that this showed convergence, and ignored the value of the common ratio and hence the mark was not awarded.

Another common incorrect response was 'the terms get smaller and smaller so the series converges to zero'.

STUDENT B

7 The terms of a sequence with first term u_1 are given by

$$u_1 = 672$$

$$u_{n+1} = -\frac{3}{4}u_n$$

7 (b) (i) State how you know that the series $u_1 + u_2 + u_3 + \ldots$ converges. **[1 mark]**

E0

Because $u_3 \div u_2 = 378 \div (-504) = -\frac{3}{4}$, $u_2 \div u_1 = (-504) \div 672 = -\frac{3}{4}$,
so $(-\frac{3}{4}) = r$, ~~There~~ so ~~it is~~ the series is ($u_1 + u_2 + u_3 + \ldots$ converges)

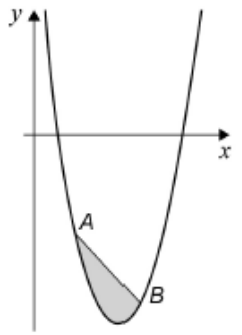
EXAMINER COMMENTARY

This example scores zero marks. The student has stated the correct common ratio, however they have not stated that its absolute value is less than one. This is an incomplete argument.
Any argument or conclusion must be fully justified if it is to be awarded any marks.

QUESTION

08 (b)

8 **(b)** The curve with equation $y = x^2 - 8x + \frac{1}{x^2} + 7$ for $x > 0$ is drawn below.



A straight line intersects the curve at the points $A(2, -4.75)$ and $B(5, -7.96)$

Find the exact area of the shaded region bounded by the curve $y = x^2 - 8x + \frac{1}{x^2} + 7$ and the line AB.

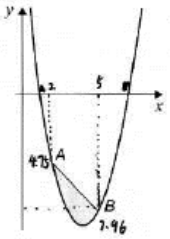
[4 marks]

MARK SCHEME

Q	Answer	Mark	Comments
8(b)	$3 \times \left(\frac{4.75 + 7.96}{2}\right)$	M1	Correct method for finding the area of the trapezium
	19.065	A1	oe Accept -19.065
	$23.7 - 19.065$	M1	oe. ft their value from Question **8(a)** provided that their 23.7 is positive and their final answer to Question **8(a)** was negative
	4.635	A1	oe
	Total	**10**	

STUDENT A

8 (b) The curve with equation $y = x^2 - 8x + \frac{1}{x^2} + 7$ for $x > 0$ is drawn below.



A straight line intersects the curve at the points $A(2, -4.75)$ and $B(5, -7.96)$

Find the exact area of the shaded region bounded by the curve $y = x^2 - 8x + \frac{1}{x^2} + 7$ and the line AB.

[4 marks]

$5-2=3$.

$-23.7-19.065=-42.765$

$\frac{1}{2}(4.75+7.96)\times 3$.

$= 19.065$

M1

A1

M0

A0

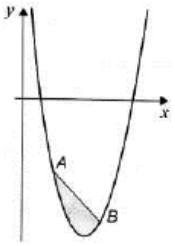
Answer -42.765

EXAMINER COMMENTARY

In this example the student has used a correct method for finding the area of the trapezium and so has gained the first method mark. They have gone on to evaluate it correctly and so have gained the first accuracy mark.
In the previous part of the question students were asked to evaluate $\int_2^5 \left(x^2 - 8x + \frac{1}{x^2} + 7\right) \mathrm{d}x$. In this case the student had correctly evaluated this integral as -23.7 but in this part they have not used it correctly and not taken the absolute value as the area between the curve and the x-axis. This has resulted in an incorrect calculation for the area of the shaded region and so the second method mark was not awarded, and as a consequence the final accuracy mark was not given.
This was not an uncommon error in this series.

STUDENT B

8 (b) The curve with equation $y = x^2 - 8x + \frac{1}{x^2} + 7$ for $x > 0$ is drawn below.



A straight line intersects the curve at the points $A(2, -4.75)$ and $B(5, -7.96)$

Find the exact area of the shaded region bounded by the curve $y = x^2 - 8x + \frac{1}{x^2} + 7$ and the line AB.

[4 marks]

$y = kx + b$

$\begin{cases} -4.75 = 2k + b \\ -7.96 = 5k + b \end{cases}$

$\therefore k = -1.07 \quad b = -2.61$

$\therefore y = -1.07x - 2.61$

$\int_2^5 (-1.07x - 2.61 - x^2 + 8x - \frac{1}{x^2} + 7)\,dx$

$= \int -0.535x^2 - 2.61x - \frac{1}{3}x^3 + 4x^2 + x^{-1} - 7x \Big|_{\text{trns}}^{\text{trns}}$

$= -\frac{247}{120} + \frac{1129}{150}$

$= \frac{927}{200} = 4.635$

Answer $S = \frac{927}{200} = 4.635$

M1

M1

A1

A1

EXAMINER COMMENTARY

This is a completely correct solution, however it is a common example of a student finding, or attempting to find, the area of the trapezium by integration.
The student has correctly found an equation for the straight line through *A* and *B*, however no credit has been given for this as the first method mark was for a correct, complete method for finding the area of the trapezium. They have, though; gone on to give a fully correct definite integral that if correctly evaluated would give the correct shaded area. Combined this shows both a correct method for finding the area of the trapezium and also the required shaded area and so has been awarded both method marks. The integral has been correctly evaluated which merits both accuracy marks even though a correct value for the area of the trapezium cannot be seen. The correct final value without incorrect working implies both accuracy marks.

Student A showed the most economical way through this problem. However, a large number of students chose to find the area of the trapezium by integration and therefore increasing their workload and also their chances of making mistakes such as incorrect signs and rounded values in the equation of the line. As a consequence, a considerable number of students failed to gain all if not any of the marks in this part of the question.

Students need to be mindful of results found previously and also more economical methods of solution.

QUESTION

09 (a)

9 A railway company bases the cost of running its trains on the speed at which they travel.

One of the company's trains travels from New York to Chicago at a steady speed of v kilometres per hour.

The cost per kilometre in dollars, C, for the journey is given by

$$C = \frac{v}{25} + \frac{100}{v}$$

9 **(a)** Find the value of v for which C is a minimum.

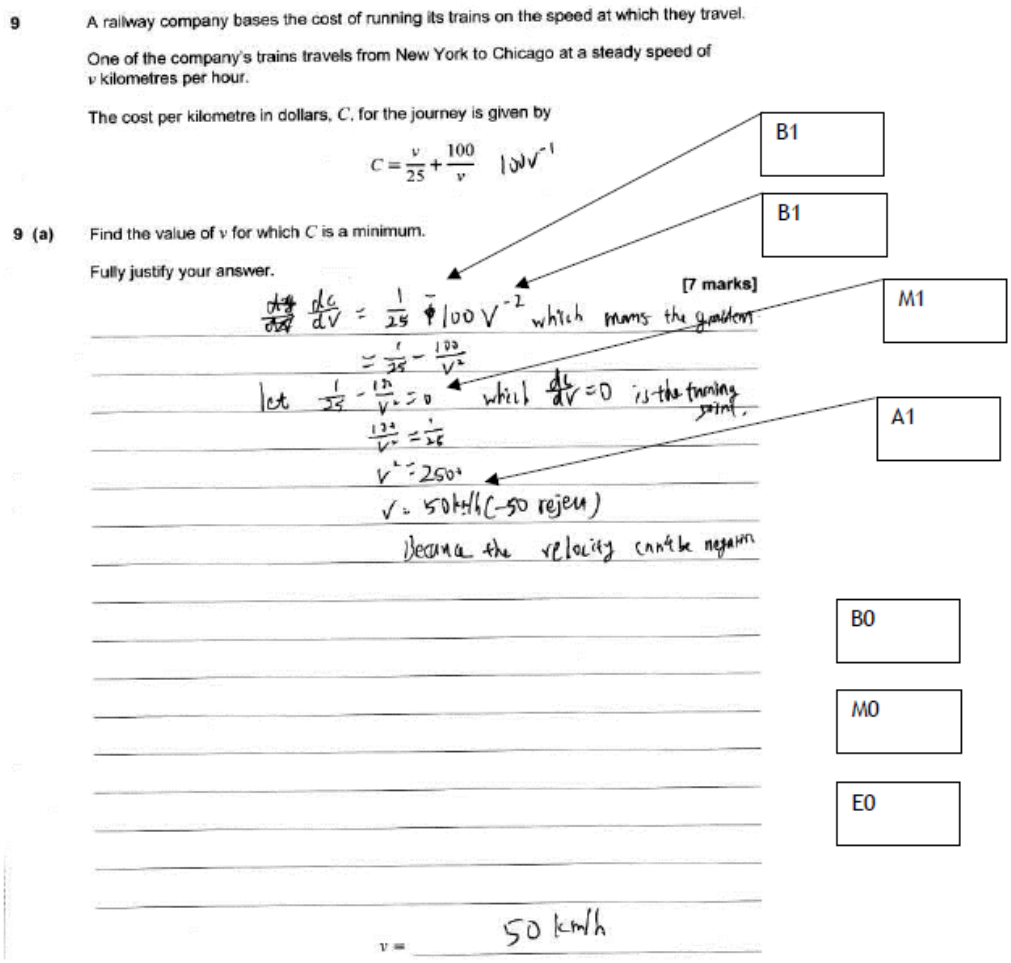
Fully justify your answer.

[7 marks]

MARK SCHEME

Q	Answer	Mark	Comments
9(a)	$C = \frac{v}{25} + 100v^{-1}$	B1	Correct conversion of $\frac{1}{v}$ to v^{-1} in a correct formula
	$\left(\frac{dC}{dv} =\right) \frac{1}{25} - \frac{100}{v^2}$	B1	Both terms correct
	$\frac{1}{25} - \frac{100}{v^2} = 0$	M1	Setting their derivative equal to zero
	$v = 50$ [km/h]	A1	CSO. Ignore $v = -50$ given in addition to $v = 50$
	$\left(\frac{d^2C}{dv^2} =\right) \frac{200}{v^3}$	B1ft	For correct differentiation of their $\left(\frac{dC}{dv}\right)$
	$v = 50 \Rightarrow \frac{d^2C}{dv^2} = \frac{1}{625}$	M1	For clear intention to evaluate the value of their second derivative at their v
	$\frac{d^2C}{dv^2} > 0$ when $v = 50 \Rightarrow C$ will be a minimum for this value of v	E1	Clear explanation resulting from completely correct working. Be convinced

STUDENT A



9 A railway company bases the cost of running its trains on the speed at which they travel.

One of the company's trains travels from New York to Chicago at a steady speed of v kilometres per hour.

The cost per kilometre in dollars, C, for the journey is given by

$$C = \frac{v}{25} + \frac{100}{v} \quad 100v^{-1}$$

9 (a) Find the value of v for which C is a minimum.

Fully justify your answer.

[7 marks]

$\frac{dC}{dv} = \frac{1}{25} - 100v^{-2}$ which means the gradient

$= \frac{1}{25} - \frac{100}{v^2}$

let $\frac{1}{25} - \frac{100}{v^2} = 0$ which $\frac{dC}{dv} = 0$ is the turning point.

$\frac{100}{v^2} = \frac{1}{25}$

$v^2 = 2500$

$v = 50$ km/h (-50 reject)

because the velocity can't be negative

$v =$ 50 km/h

B1

B1

M1

A1

B0

M0

E0

EXAMINER COMMENTARY

In this part of the question for the first mark the student should have shown the correct conversion of $\frac{1}{v}$ to v^{-1} in a correct formula for *C*. This student has not shown this; however, they have given the correct first derivative of *C* which implies the first mark. This correct derivative also gains the second mark. They have set their derivative to equal zero and so they have been awarded the first method mark, going on to get the fourth mark for solving this equation and obtaining the correct value of v.

The final three marks were for justifying that it is a minimum value, the first of which for correctly differentiating their first derivative. The second for attempting to evaluate the value of their second derivative for their value of v. Finally, the third for a conclusion stating that it is a minimum since the value of the second derivative is positive. This final mark would only be awarded if their explanation resulted from completely correct working. This student has not attempted a justification and so these last three marks have not been awarded.

In general, the answer to any question concerning maximum or minimum values should include the student fully justifying that it is a maximum or a

STUDENT B

9 A railway company bases the cost of running its trains on the speed at which they travel.

One of the company's trains travels from New York to Chicago at a steady speed of v kilometres per hour.

The cost per kilometre in dollars, C, for the journey is given by

$$C = \frac{v}{25} + \frac{100}{v}$$

9 (a) Find the value of v for which C is a minimum.

Fully justify your answer.

[7 marks]

~~$v = \frac{5}{t}$~~ ~~$5 = vt$~~

when $\frac{v}{25} + \frac{100}{v}$ is a minimum.

v	C
5	20.2
15	7.27
25	5
35	4.26
45	4.02
50	4
55	4.02
65	4.14
75	4.33
85	4.58
95	4.85
100	5
110	5.31

∴ when $v = 50$, C is a minimum.

$v =$ 50.

EXAMINER COMMENTARY

In this part of the question the student did not gain any marks. They have arrived at the correct value for v, however they should have used differentiation to obtain this.
Any solution attempted either by using trial and improvement or listing a table of values will not gain credit in an optimisation problem of this type.

FURTHER GUIDANCE AND CONTACTS

You can contact the subject team directly at **maths@oxfordaqaexams.org.uk**

Please note: We aim to respond to all email enquiries within two working days.

Our UK office hours are Monday to Friday, 8am – 5pm local time.

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