OXFORD
INTERNATIONAL
AQA EXAMINATIONS

INTERNATIONAL A-LEVEL FURTHER MATHEMATICS

(9665)

Example responses with commentary: FM05, Unit FM2

For teaching from September 2017 onwards

INTRODUCTION

This guide includes student's responses to questions from the June 2019 International A-level Further Mathematics (9665) FM05, Unit FM2.

The questions are presented with mark scheme, student responses and commentary from the Lead Examiner.

ASSESSMENT OBJECTIVES

The exams will measure how students have achieved the following assessment objectives:

- AO1: Recall and select knowledge of mathematical facts, concepts, models and techniques required to solve problems in a variety of contexts.
- AO2: Construct rigorous mathematical arguments and proofs through use of precise statements, mathematical manipulation, logical deduction, modelling assumptions and justifications to solve structured and unstructured problems, and to deduce, interpret and communicate results.

KEY TO MARK SCHEME ABBREVIATIONS

M	Mark is for method
m	Mark is dependent on one or more m marks and is for method
A	Mark is dependent on m or m marks and is for accuracy
B	Mark is independent of m or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
−x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied

SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

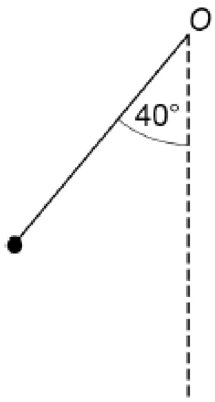
EXAMPLE RESPONSES

QUESTION 1

1 A particle, of mass 0.1 kg, is attached to one end of a light inextensible string of length 0.8 metres.

The other end of the string is attached to a fixed point *O*.

The particle is released from rest with the string taut and at an angle of 40° to the vertical through *O*.



Assume that there are no resistance forces acting on the particle.

1 (a) Find the speed of the particle when it is directly below *O*.

[3 marks]

1 (b) Find the tension in the string when the particle is directly below *O*.

[3 marks]

MARK SCHEME

Q	Answer	Mark	Comments
1 (a)	$0.1 \times 9.8 \times 0.8(1 - \cos 40°) = \frac{1}{2} \times 0.1v^2$ $v = \sqrt{15.68(1 - \cos 40°)} = 1.9\ \mathrm{m\,s^{-1}}$	M1 A1 A1 **3**	M1: GPE found using either cos40° or sin 40° A1:Correct energy equation A1: Correct speed. AWRT 1.9
1 (b)	$T - 0.1 \times 9.8 = 0.1 \times \frac{15.68(1 - \cos 40°)}{0.8}$ $T = 1.4\ \mathrm{N}$	M1 A1 A1 **3**	M1: Equation of motion using radial acceleration formula. Allow their speed from part (a). A1: Correct three term equation of motion. A1: Correct tension. AWRT 1.4
	Total	**6**	

STUDENT A

RESPONSE

1 (a) Find the speed of the particle when it is directly below O.

[3 marks]

GPE → KE $mgh = \frac{1}{2}mv^2$ $2gh = v^2$

$\cancel{0.1g(1-\cos 40) = \frac{1}{2}\times 0.1 V^2}$ $V = \sqrt{2\times 9.8\times 0.8(1-\cos 40)}$

$\cancel{V = 2.14 \text{ m/s.}}$ $V = 1.92$ m/s

Answer 1.92 m s^{-1}

1 (b) Find the tension in the string when the particle is directly below O.

[3 marks]

$T = mg + \frac{mv^2}{r}$ 1.44·5.

$= 0.1g + \frac{2mgh}{r}$

$= 0.1g + \frac{0.2g\times 0.8(1-\cos 40)}{0.8}$

$= 0.1g + 0.2g(1-\cos 40)$

$= 1.44$ N

Answer 1.44 N

COMMENTARY

In part (a), the student made an incorrect start, omitting the length of the string but realized this error and then completed the question correctly. Note that the student cancelled the mass at the start of their working and never actually used the value of 0.1.

In part (b), the student produced a clear equation to illustrate the approach to be taken. Note that the student substituted an expression for the speed rather than using their value of 1.92 to ensure that an accurate final answer was obtained.

The student did not follow the instruction to give their answers to 2 significant figures but was not penalized for this in this question.

MARKS AWARDED: The student was awarded 6 marks out of a possible 6.

STUDENT A

RESPONSE

1 (a) Find the speed of the particle when it is directly below *O*.

[3 marks]

Gravitational potential energy:

$mgh = 0.1 \times 9.8 \times (0.8 - 0.8 \cdot \cos 40^\circ)$

$= 0.183$

$\cancel{m}\ \Delta mgh = \Delta K.E$

$0.183 = \frac{1}{2} \cdot 0.1 \cdot v^2$

$v = 1.9$

Answer 1.9 m s^{-1}

1 (b) Find the tension in the string when the particle is directly below O.

[3 marks]

$$T = mg - \frac{mv^2}{r}$$

$$= 0.1 \times 9.8 - \frac{0.1 \cdot (1.9)^2}{0.8}$$

$$= 0.52$$

Answer 0.52 N

COMMENTARY

In part (a), the student provides a very clear, well set out solution.

In part (b), the student includes the correct terms in their equations but makes sign errors, gaining only 1 of the available marks. Also note that this student uses their rounded answer for the speed from part (a).

This student did give all of the final answers correct to two significant figures.

MARKS AWARDED: The student was awarded 4 marks out of a possible 6.

QUESTION 2

2 A bungee jumper, of mass 75 kg, is attached to one end of an elastic rope of natural length 20 metres.

The other end of the elastic rope is fixed to a bridge.

The bungee jumper steps off the bridge at the point where the rope is fixed and falls vertically downwards.

During the bungee jump the maximum length of the elastic rope is 50 metres.

2 (a) Find the modulus of elasticity of the elastic rope.

[3 marks]

2 (b) Find the maximum speed of the bungee jumper during the motion.

[7 marks]

MARK SCHEME

Q	Answer	Mark	Comments
2 (a)	$\frac{\lambda \times 30^2}{2 \times 20} = 75 \times 9.8 \times 50$ $\lambda = \frac{1470000}{900} = \frac{4900}{3} = 1600$ N	M1 B1 A1 **3**	M1: Correct GPE used in two term energy equation. B1: Correct initial EPE. A1: Correct modulus. AWRT 1600
2 (b)	Max speed when: $\frac{4900 \times e}{3 \times 20} = 75 \times 9.8$ $e = 9\text{ m}$ Speed given by: $29 \times 75 \times 9.8 = \frac{1}{2} \times 75v^2 + \frac{4900 \times 9^2}{2 \times 3 \times 20}$ $v = \sqrt{480.2} = 22\text{ m s}^{-1}$	M1 B1 A1 M1A1 A1 A1 **7**	M1: Tension and weight equated. B1: Correct tension A1: Correct extension for equilibrium. M1: Energy equation with at least two terms correct with any signs M1: Energy equation with correct terms and any signs. A1: Correct equation. A1: Correct speed. AWRT 22
2 (c)	No air resistance. Bungee Jumper is a particle	B1 **1**	B1: Two appropriate assumptions.
	Total	**11**	

STUDENT A

RESPONSE

2 (a) Find the modulus of elasticity of the elastic rope.

[3 marks]

$Ep = EpE$

$mgh = \frac{1}{2}kl^2$

$75 \times 9.8 \times 50 = \frac{1}{2} \cdot k \times 30^2$

$k = 81.7$ N/m.

$\therefore$ modulus $= 81.7 \times 20$

$= 1634$ N

Answer 1634 N

2 (b) Find the maximum speed of the bungee jumper during the motion.

[7 marks]

~~$Ep = EpE + Ek$~~

~~$\therefore$ When $EpE = 0$ Ek is maximum.~~

~~$V^2 = u^2 + 2as.$~~

~~$V^2 = 2 \times 9.8 \times 20$~~

~~$V = 19.8$ m/s.~~

maximum speed occurs when $a = 0$

$\therefore$ Weight = Tension.

$75 \times 9.8 = 81.7 \, \Delta l$

$\Delta l = 9$ m.

$Ep = EpE + Ek$

$75 \times 9.8 \times (20+9) = \frac{1}{2} \times 81.7 \times 9^2 + \frac{1}{2} \times 75 \times v^2$

$V = 21.9$ m/s.

Answer ~~19.8~~ 21.9 m s^{-1}

2 (c) State **two** key assumptions that you made to obtain the answers in parts **(a)** and **(b)**.

[1 mark]

There is no air resistance.

Assume the jumper as a particle.

COMMENTARY

In part (a), the student forms an equation to find the stiffness of the rope and then obtains the modulus of elasticity from this.

In part (b), the student begins by stating that the acceleration must be zero and uses this to find the corresponding length of the rope. The student then applies conservation of energy to find the required speed.

Two valid assumptions are stated in part (c).

The student did not follow the instruction to give their answers to 2 significant figures but was not penalized for this in this question.

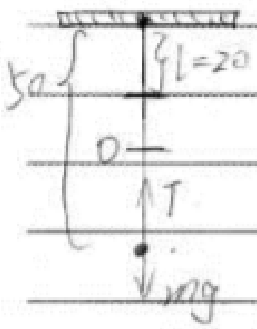
MARKS AWARDED: The student was awarded 11 marks out of a possible 11.

STUDENT B

RESPONSE

2 (a) Find the modulus of elasticity of the elastic rope.

[3 marks]



When at the lowest point, $T = mg$.

So $\frac{\lambda e}{l} = mg$.

$\frac{\lambda \times (50-20)}{20} = 75 \times 9.8$.

$\lambda = 490$ N.

Answer 490. N

2 (b) Find the maximum speed of the bungee jumper during the motion.

[7 marks]

When it's passing point O, it reaches the maximum speed. When it at the lowest point, it only has elastic potential energy and at O, it has g.p.e. k.e, and smaller e.p.e

So $E_{max} = \frac{490 \times (50-20)^2}{2 \times 20} = \cancel{900J}\ 11025$

E at O: $\frac{1}{2}mv^2 + mgh + \frac{\lambda x'^2}{l} = 900J$.

$\frac{1}{2} \times 75 \times v^2 + 75 \times 9.8 \times 15 + \frac{490 \times 15^2}{2 \times 20} = \cancel{900}\ 11025$.

$v > 0$ $v = 8.6\ ms^{-1}$.

Answer 8.6 m s^{-1}

2 (c) State **two** key assumptions that you made to obtain the answers in parts **(a)** and **(b)**.

[1 mark]

① There is no air resistance.

② The rope obeys Hooke's law.

COMMENTARY

This student does not adopt the correct strategies in either of parts (a) and (b). In part (a) the student uses energy rather than considering forces and in part (b) considers energy rather than using the forces to find an extension first. The student does not seem to be sufficiently aware of the properties of this type of motion.

The student did give two valid assumptions in part (c).

MARKS AWARDED: The student was awarded 1 mark out of a possible 11.

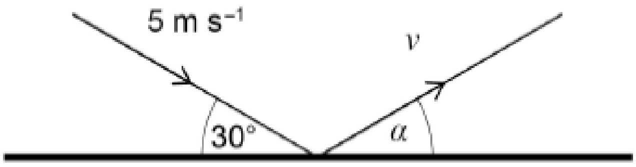
QUESTION 3

3 A disc, of mass 0.5 kg, is moving on a smooth horizontal surface, when it hits a smooth wall.

When it hits the wall, the disc is moving at 5 m s^{-1} and its velocity makes an angle of 30° with the wall.

The coefficient of restitution between the disc and the wall is 0.4

The disc rebounds with a speed of v m s^{-1} at an angle α to the wall, as shown in the diagram.



3 (a) Find the value of α.

[7 marks]

3 (b) Find the value of v.

[3 marks]

3 (c) Find the magnitude of the impulse on the disc.

[3 marks]

MARK SCHEME

Q	Answer	Mark	Comments
3 (a)	$5\cos 30° = v\cos\alpha$ $0.4 \times 5\sin 30° = v\sin\alpha$ $\frac{\sin\alpha}{\cos\alpha} = \frac{2\sin 30°}{5\cos 30°}$ $\tan\alpha = \frac{2\sqrt{3}}{15}$ $\alpha = 13.0039°$ $\alpha = 13°$	M1A1 M1A1 M1A1 A1 **7**	M1: Equation for motion parallel to wall. A1: Correct equation. M1: Equation for motion perpendicular to wall. Must include 0.4 A1: Correct equation. M1: Expression for tan α A1: Correct expression. A1: Correct angle. AWRT 13°
3 (b)	$v = \frac{5\cos 30°}{\cos\alpha}$ $v = 4.4$ Or $v = \frac{2\sin 30°}{\sin\alpha}$ $v = 4.4$	M1A1 A1 (M1A1) (A1) **3**	M1: Equation with v as the only unknown. A1: Correct equation. A1: Correct value for v. AWRT 4.4
3 (c)	$I = 0.5 \times 4.44\sin 13° - 0.5(-5\sin 30°)$ $I = 1.7 \text{ N s}$	M1A1 A1 **3**	M1: Impulse equation with correct values and any signs. A1: Correct equation. A1: Correct impulse AWFW [1.7, 1.8]
	Total	**13**	

STUDENT A

RESPONSE

3 (a) Find the value of α.

[7 marks]

$$\begin{cases} V\cos\alpha = 5\times\cos 30° \\ \cancel{5}V\sin\alpha = 5\sin 30°\times 0.4 \end{cases}$$

$$V = \frac{\cancel{5}\cdot 2\sin 30}{\sin\alpha}$$

$$\frac{2\sin 30}{\sin\alpha}\cdot\cos\alpha = 5\cos 30°$$

$$2\frac{\cos\alpha}{\sin\alpha} = \frac{5\cos 30°}{\sin 30°}$$

$$\tan\alpha = \frac{2\sin 30°}{5\cos 30}$$

$$= \frac{2\sqrt{3}}{15}$$

$$\alpha = 13°$$

Answer 13°

3 (b) Find the value of v.

[3 marks]

$V \cos 13° = 5 \times \cos 30°$

$v = 4.4$

Answer 4.4

3 (c) Find the magnitude of the impulse on the disc.

[3 marks]

~~I = m~~ ~~mv~~

$= \cancel{m \times 4.4}\ \cancel{0.5 \times (4.4 - 5\ 5 - 4.4)}$

$= \cancel{0.3 Ns}$

~~I =~~ $I = 0.5\ (5 \sin 30° + 4.4 \times \cancel{\sin 30} \sin 13°)$

$= 1.7 Ns$

Answer ~~0.3~~ 1.7 N s

COMMENTARY

This student formulated correct equations for the velocity of the disc before and after the collision with the wall and manipulated these to obtain the required angle in part (a) and the value of v in part (b).

It is interesting to see that the student initially worked with the velocities, rather than the components of the velocities for part (c).

MARKS AWARDED: The student was awarded 13 marks out of a possible 13.

STUDENT B

RESPONSE

3 (a) Find the value of α.

[7 marks]

~~Vertical $5\sin 30^\circ = v\sin\alpha$~~

~~Horizontal $5\cos 30^\circ = v\cos\alpha$~~

~~$\frac{2}{5} = \frac{v\sin\alpha}{-5\cos 30^\circ}$~~

~~$\sqrt{3} = v\sin\alpha$~~

The wall is smooth so ~~both~~ vertical component does not change.

So $5\sin 30^\circ = v\sin\alpha$

$\frac{2}{5} = \frac{v\cos\alpha}{-5\cos 30^\circ}$

$v\cos\alpha = \sqrt{3}$.

$\frac{\sin\alpha}{\cos\alpha} = \frac{5\sqrt{3}}{6}$.

$\alpha = 55.3^\circ$

Answer 55.3°

3 (b) Find the value of v.

[3 marks]

$$V = \frac{\sqrt{3}}{\cos 55.3^\circ}$$

$$= 3.04$$

Answer 3.04.

3 (c) Find the magnitude of the impulse on the disc.

[3 marks]

$$I = \cancel{mu - mv} \quad mv - mu$$

$$= 0.5(3.04 - 5)$$

$$= -0.98$$

Answer -0.98. N s

COMMENTARY

In part (a), the opening statement by the student shows some understanding of the ideas required for this question. However, the student resolves incorrectly, but consistently. As a result, the student gains the three method marks that are available. Mixing up the use of sine and cosine in questions like this is a fairly common error.

In part (b), the student uses the same incorrect equations as in part (a) and is only able to obtain the one method mark.

In part (c), the student uses the velocities and not the components of the velocities. This is similar in principle to the initial approach of Student A. It is also interesting to note that this student gives a negative value for their final answer, which should be positive as the question asks for the magnitude of the impulse.

MARKS AWARDED: The student was awarded 4 marks out of a possible 13.

QUESTION 4

4 A particle, of mass m, moves on a horizontal line subject to a resistance force of magnitude kv, where k is a constant and v is the speed of the particle at time t.

When $t = 0$, the particle is at the origin and has speed U.

4 (a) Show that

$$m\frac{\mathrm{d}v}{\mathrm{d}x} = -k$$

where x is the displacement of the particle at time t.

[2 marks]

4 (b) Show that the particle travels no further than $\frac{mU}{k}$ from the origin.

[4 marks]

4 (c) Find, in terms of k and m, the time taken for the speed of the particle to decrease to $\frac{U}{2}$

[7 marks]

MARK SCHEME

Q	Answer	Mark	Comments
4 (a)	$mv\dfrac{\mathrm{d}v}{\mathrm{d}x} = -kv$ $m\dfrac{\mathrm{d}v}{\mathrm{d}x} = -k$	M1 A1 **2**	M1: Differential equation with correct terms and any signs. A1: Simplified correct differential equation.
4 (b)	$mv = -kx + c$ Using $x = 0, v = U$ $mU = c$ Using $v = 0$ $0 = -kx + mU$ $x = \dfrac{mU}{k}$	M1 A1 M1 A1 **4**	M1: Integrating their equation. Condone omission of *c*. A1: Correct integration. M1: Initial values used to find *c*. A1: Correct value of *c* and correct final answer from correct working.
4 (c)	$m\dfrac{dv}{dt} = -kv$ $\dfrac{m}{v}\dfrac{dv}{dt} = -k$ $m\ln(v) = -kt + c$ Using t$= 0, v = U$ $m\ln(U) = c$ Using $v = \frac{U}{2}$ $m\ln\left(\dfrac{U}{2}\right) = -kt + m\ln(U)$ $t = \dfrac{m}{k}\ln(2)$	M1 M1 A1 M1 A1 M1 A1 **7**	M1: Differential equation with correct terms and any signs. M1: Integrating their equation. Condone omission of *c*. A1: Correct integrals. M1: Initial values used to find *c*. A1: Correct value of *c*. M1: Substitutes $\frac{U}{2}$. A1: Correct time.
	Total	**13**	

STUDENT A

RESPONSE

4 (a) Show that

$$m\frac{\mathrm{d}v}{\mathrm{d}x} = -k$$

where x is the displacement of the particle at time t.

[2 marks]

$m \cdot \dfrac{dv}{dx} = m \cdot \dfrac{dv}{dt} \cdot \dfrac{dt}{dx} = ma \cdot \dfrac{1}{v}$

$\because F = ma = -kv$

$\therefore ma \cdot \frac{1}{v} = -kv \cdot \frac{1}{v} = -k$

$\therefore m\dfrac{dv}{dx} = -k$

4 (b) Show that the particle travels no further than $\dfrac{mU}{k}$ from the origin.

[4 marks]

$\cancel{\because \dfrac{dv}{dx} = -\dfrac{k}{m}}$

$\cancel{\therefore dv = -\dfrac{k}{m} dx}$

$\cancel{\int 1\, dv = \int -\dfrac{k}{m}\, dx}$

$\cancel{\therefore v = -\dfrac{k}{m} x + c}$

$\cancel{\because t=0,\ x=0,\ v=U}$

$\cancel{\therefore c = U}$

$\cancel{\therefore v = -\dfrac{k}{m} x + U}$

$\cancel{\therefore \dfrac{dx}{dt} = -\dfrac{k}{m} x + U}$

$\therefore m\,dv = -k\,dx$

$\int m\,dv = \int -k\,dx$

$mv = -kx + c$

$\because t=0,\ x=0,\ v=U$

$\therefore c = mU$

$\therefore kx = mU - mv$

$x = \dfrac{mU}{k} - \dfrac{m}{k} \cdot v$

$\because v$ is positive

$\therefore x \leq \dfrac{mU}{k}$

$\therefore$ the particle travels no further than $\dfrac{mU}{k}$.

4 (c) Find, in terms of k and m, the time taken for the speed of the particle to decrease to $\dfrac{U}{2}$

[7 marks]

$\because\ mv = -kx + mU$

$\therefore\ m\dfrac{dx}{dt} = -kx + mU$

$\therefore\ \dfrac{1}{-kx+mU}\,dx = \dfrac{1}{m}\,dt$

$\cancel{\ln|-kx+mU|}$

$\therefore\ \int \dfrac{1}{-kx+mU}\,dx = \dfrac{1}{m}\,dt$

$\therefore\ -\ln|-kx+mU| = \dfrac{t}{m} + c_1$

$\because\ t=0,\ x=0$

$\therefore\ c_1 = -\ln|mU|$

$\therefore\ t = m\ln|mU| - m\ln|-kx+mU|$

$\because\ ma = -kv$

$\therefore\ m\dfrac{dv}{dt} = -kv$

$\therefore\ \dfrac{1}{v}\,dv = -\dfrac{k}{m}\,dt$

$\int \dfrac{1}{v}\,dv = -\dfrac{k}{m}\,dt$

$\ln|v| = -\dfrac{k}{m}t + c$

~~$\because\ t=0,\ v=U$~~

~~$\therefore\ c = \ln U$~~

~~$\therefore\ \ln|v| = -\dfrac{k}{m}$~~

$\therefore\ v = e^{-\frac{k}{m}t+c}$

$\because\ t=0,\ v=U$

$\therefore\ e^{c} = U$

$\therefore\ v = U\cdot e^{-\frac{k}{m}t}$

$v = \dfrac{U}{2}$

$\dfrac{U}{2} = U\cdot e^{-\frac{k}{m}t}$

$\therefore\ -\dfrac{k}{m}t = \ln\dfrac{1}{2} = -\ln 2$

$\therefore\ \dfrac{k}{m}t = \ln 2$

$t = \dfrac{m}{k}\ln 2$

Answer $\dfrac{m}{k}\ln 2$

COMMENTARY

This student completes all parts of the question with detailed mathematical arguments. It is interesting to note that the incorrect attempts have been clearly crossed through and replaced with correct working.

In part (a), the student clearly indicates how the product of the derivatives has been used.

In part (b), the student shows all stages of the integration, the value of the constant of integration and the condition required to obtain printed answer is stated clearly.

Similarly, in part (c), all stages of the integration are shown and the value of the constant of integration stated. The velocity given in the question is then used to find the required time.

MARKS AWARDED: The student was awarded 13 marks out of a possible 13.

STUDENT B

RESPONSE

4 (a) Show that

$$m\frac{\mathrm{d}v}{\mathrm{d}x} = -k$$

where x is the displacement of the particle at time t.

[2 marks]

$\cancel{f = F = -kv = ma}$ $F = -f = -kv = ma.$

$m \cdot v\frac{dv}{dx} = -kv$

$m\frac{dv}{dx} = -k$

4 (b) Show that the particle travels no further than $\frac{mU}{k}$ from the origin.

[4 marks]

$\cancel{m =}$ $m\frac{dv}{dx} = -k$

$\int m\,dv = \int -k\,dy$

$mv = -kx + c.$

when $t = 0$ $x = 0$ $v = U$

$mU = c.$

$\therefore mv = -kx + mU$

$mv - mU = -kx$

when $v = 0$

$-mU = -kx$

$x = \frac{mU}{k}$

4 (c) Find, in terms of k and m, the time taken for the speed of the particle to decrease to $\frac{U}{2}$

[7 marks]

$v = \frac{dx}{dt}$.

When $v = \frac{U}{2}$

$\frac{U}{2} = \frac{dx}{dt}$.

$\int \frac{U}{2} dt = \int dx$

$\frac{U}{2} t = x$

$\frac{U}{2} t = \frac{mU}{k}$

$t = \frac{2m}{k}$

COMMENTARY

In part (a), this students uses the fact that $a = v\frac{dv}{dx}$ and shows the minimum amount of working needed to obtain the marks for this part of the question, in contrast to Student A who shows how this result can be obtained.

In part (b), the student provides a good complete solution.

In part (c), the student tries to use $\frac{dx}{dt}$, but equates this to a constant and obtains a very simple but incorrect result. Without forming a second differential equation, this student was unable to gain any of the marks for this part of the question.

MARKS AWARDED: The student was awarded 6 marks out of a possible 13.

QUESTION 5

5 **In this question, give your final answer to each part to three significant figures.**

A sphere, of mass 0.5 kg, is attached to one end of a spring, of natural length 50 cm.

The other end of the spring is attached to a fixed point, *O*.

The sphere is pulled down and released from rest at a point directly below *O*.

The sphere performs simple harmonic motion moving between two points *A* and *B*, which are 10 cm apart, with *A* above *B*.

During this motion, the maximum speed of the sphere is 1.5 m s^{-1}

5 (a) Find the period of the motion.

[3 marks]

5 (b) Find the stiffness of the spring.

[3 marks]

5 (c) Find the maximum length of the spring during the motion.

[4 marks]

5 (d) Find the speed of the sphere when the spring is at its natural length.

[3 marks]

MARK SCHEME

Q	Answer	Mark	Comments
5 (a)	$1.5 = 0.05\omega$ $\omega = 30$ $\text{Period} = \frac{2\pi}{30} = 0.209\text{ s}$	M1 A1 A1 **3**	M1: Max speed used to form an equation to find ω. A1: Correct ω. A1: Correct period. AWRT 0.21
5 (b)	As SHM $0.5\ddot{x} = -kx$ $2k = 30^2$ $k = \frac{900}{2} = 450\text{ N m}^{-1}$	M1 A1 A1 **3**	M1: Differential equation in terms of k. A1: Correct equation for k. A1: Correct k.
5 (c)	In equilibrium $0.5 \times 9.8 = 450e$ $e = \frac{4.9}{450} = 0.0109\text{ m}$ $\text{Max extension} = 0.0109 + 0.05$ $= 0.0609\text{ m}$ $\text{Max length} = 0.0609 + 0.5 = 0.561\text{ m}$	M1 A1 A1 A1 **4**	M1: Equation to find extension in equilibrium. A1: Correct extension. A1: Correct maximum extension. A1: Correct maximum length. AWRT 0.56
5 (d)	$v^2 = 30^2(0.05^2 - 0.0109^2)$ $v = 1.46\text{ m s}^{-1}$	M1A1 A1 **3**	M1: Use of SHM equation with correct values. The terms x and a may be interchanged. A1: Correct equation. A1: Correct speed. AWRT 1.5
	Total	**13**	

STUDENT A

RESPONSE

5 (a) Find the period of the motion.

[3 marks]

$V_{max} = \omega A$, $A = \frac{10}{2} = 5\text{cm} = 0.05\text{m}$

$1.5 = 0.05\left(\frac{2\pi}{T}\right)$ $\therefore T = \frac{0.05(2\pi)}{1.5}$

$= \frac{1}{15}\pi = 0.209 \text{ s}$

Answer 0.209 seconds

5 (b) Find the stiffness of the spring.

[3 marks]

(e = extension) $\frac{\lambda e}{l} = mg$ (Initially, before pull), $\frac{\lambda(e+x)}{l} = mg - ma$

$\therefore \frac{\lambda e}{l} + \frac{\lambda x}{l} - mg = -ma \Rightarrow \frac{\lambda e}{l} - \frac{\lambda e}{l} + \frac{\lambda x}{l} = -ma$

$\therefore a = -\frac{\lambda}{ml} x$ $\therefore$ SHM w/ $\omega = \sqrt{\frac{\lambda}{ml}}$

$\frac{2\pi}{\left(\frac{\pi}{15}\right)} = \sqrt{\frac{\lambda}{(0.5)(0.5)}}$ $\therefore \lambda = (0.5)(0.5)(30)^2 = 225$

$\therefore k = \frac{\lambda}{l} = \frac{225}{0.5} = 450 \text{ Nm}^{-1}$

Answer 450 N m^{-1}

5 (c) Find the maximum length of the spring during the motion.

[4 marks]

Without oscillation, $kx = mg \Rightarrow x = \frac{0.5g}{450}$

$= 0.011$ m (extention in equilibrium).

Dist. from there to B $= 0.05$ m

$\therefore$ Total spring length$_{max}$ $= 0.5 + 0.011 + 0.05$

$= 0.561$ m

Answer 0.561 m

5 (d) Find the speed of the sphere when the spring is at its natural length.

[3 marks]

$V = \omega\sqrt{A^2 - x^2}$, ~~Equilibrium at 0.5+0.011 = 0.511m~~

$\therefore$ ~~$x = 0.511 - 0$~~, $x = 0.011$ m

$\therefore V = \frac{2\pi}{\left(\frac{\pi}{15}\right)}\sqrt{0.05^2 - 0.011^2} = 1.46$ ms^{-1}

Answer 1.46 m s^{-1}

COMMENTARY

In part (a), students were expected to identify the value of ω, but this student kept this as an unknown and found the period correctly without identifying the value of ω.

In part (b), the student provided a detailed argument to find an expression for the acceleration of the sphere. It worth noting how this student defines the value of *e* used in their working. Interestingly, this student first finds the modulus of elasticity and then the stiffness.

In part (c), the student first finds the extension in the equilibrium position and then uses this to find the maximum length.

In part (d), the student uses their distance from part (c) and substitutes this into the SHM equation to find the correct speed.

This student gives all of their answers to three significant figures as instructed in the question.

MARKS AWARDED: The student was awarded 13 marks out of a possible 13.

STUDENT B

RESPONSE

5 (a) Find the period of the motion.

[3 marks]

$V_{max} = \omega A$

$1.5 = \omega \times 0.05$

$\omega = 30$

$T = \frac{2\pi}{\omega}$

$= 0.2s$

Answer 0.2 seconds

5 (b) Find the stiffness of the spring.

[3 marks]

$T = 2\pi\sqrt{\frac{m}{k}}$

$0.2 = 2\pi\sqrt{\frac{0.6}{k}}$

$k = 493 Nm^{-1}$

Answer 493 N m^{-1}

5 (c) Find the maximum length of the spring during the motion.

[4 marks]

$T = kx = mg$

$x = \frac{0.5 \times 4.9}{\cancel{1}493}$

$= 9.9 \times 10^{-3}$ m

$L = 9.9 \times 10^{-3} \times 0.05 + 0.5$

$= 0.56$

Answer ~~9.9×10^{-3}~~ 0.56 m

5 (d) Find the speed of the sphere when the spring is at its natural length.

[3 marks]

$V = \pm \omega \sqrt{A^2 - x^2}$

$V = 30 \sqrt{0.05^2 - (9.9 \times 10^{-3})^2}$

$= 1.47$ m s^{-1}

Answer 1.47 m s^{-1}

COMMENTARY

In part (a), the student obtains the correct value for ω and uses the correct approach to find the period. However, as the student only gives their final answer to one significant figure the final accuracy mark for this part was not awarded.

In part (b), the student correctly quotes the formula for the period and substitutes values to find the stiffness. The approach is correct, but because the rounded answer from part (a) is used the student is unable to obtain the correct final answer and does not gain the final accuracy mark.

In part (c), the student again uses the correct approach, but is hampered by their incorrect stiffness. In this case the student is given credit for the correct approach and gains three of the four available marks.

In part (d), the student obtains an acceptable answer but this seems to follow from incorrect working as there is a spurious 2 inside their square root.

This student has suffered the consequences of rounding their answer to part (a) to an unacceptable level and then using this rounded value in later parts of the questions. Students should be encouraged to use answers that have not been extensively rounded in later calculations within a question.

MARKS AWARDED: The student was awarded 9 marks out of a possible 13.

QUESTION 6

6 A ball is thrown from a point O on a plane which is inclined at an angle of 30° to the horizontal.

The ball is thrown up the plane with velocity U m s^{-1} at an angle θ to the inclined plane.

The ball travels in a vertical plane containing a line of greatest slope of the inclined plane.

The velocity of the ball is perpendicular to the plane when it first hits the plane.

6 (a) Show that

$$\tan\theta = \frac{\sqrt{3}}{2}$$

[7 marks]

6 (b) Find, in terms of U, the speed at which the ball first hits the plane.

[4 marks]

MARK SCHEME

Q	Answer	Mark	Comments
6 (a)	$y = U\sin\theta t - \frac{1}{2}g\cos30°t^2$	M1A1	M1: Equation for distance from the plane. Allow sign / angle errors. A1: Correct equation.
	$0 = U\sin\theta t - \frac{\sqrt{3}}{4}gt^2$	M1	M1: Solving *their* quadratic for t
	$t = \frac{4U\sin\theta}{g\sqrt{3}}$	A1	A1: Correct time.
	$\dot{x} = U\cos\theta - g\sin30°t$	M1	M1: Equation for velocity parallel to the plane. Allow sign / angle errors.
	$0 = U\cos\theta - \frac{g}{2} \times \frac{4U\sin\theta}{g\sqrt{3}}$	A1	A1: Correct equation.
	$\sqrt{3}\cos\theta = 2\sin\theta$ $\tan\theta = \frac{\sqrt{3}}{2}$	A1	A1: AG, CSO.
		7	
6 (b)	$\dot{y} = U\sin\theta - g\cos30°t$	M1	M1: Equation for velocity perpendicular to the plane. Allow sign / angle errors. A1: Correct equation. A1: Correct velocity A1: Correct speed.
	$\dot{y} = U\sqrt{\frac{3}{7}} - \frac{\sqrt{3}}{2}g \times \frac{4U}{g\sqrt{3}} \times \sqrt{\frac{3}{7}}$	A1	
	$\dot{y} = -U\sqrt{\frac{3}{7}}$	A1	
	Speed $= U\sqrt{\frac{3}{7}} = \frac{U\sqrt{21}}{7} = 0.65U$	A1	
		4	
	Total	**11**	

STUDENT A

RESPONSE

6 (a) Show that

$$\tan\theta = \frac{\sqrt{3}}{2}$$

[7 marks]

$V_y = u_y + a_y t$

$= U\sin\theta \; - g\cos 30^\circ t$

$V_x = u_x + a_x t$

$= U\cos\theta - g\sin 30^\circ t$

~~on tin~~

$\therefore V_y = 0$

$t = \dfrac{U\sin\theta}{g\cos 30^\circ}$

$\therefore t_{total} = \dfrac{2U\sin\theta}{g\cos 30^\circ}$

$\therefore V_x = U\cos\theta - g\sin 30^\circ \cdot \dfrac{2U\sin\theta}{g\cos 30^\circ} = U\cos\theta - \dfrac{\sqrt{3}}{3} \cdot 2U\sin\theta$

$\because$ velocity perpendicular to the plane

$\therefore V_x = 0$

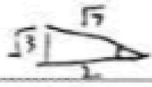
$\therefore U\cos\theta - \dfrac{2\sqrt{3}}{3} U\sin\theta = 0$

$\dfrac{2\sqrt{3}}{3} \sin\theta = \cos\theta$

$\tan\theta = \dfrac{\sqrt{3}}{2}$

6 (b) Find, in terms of U, the speed at which the ball first hits the plane.

[4 marks]



$\therefore \; V = V_y = U\sin\theta - g\cos 30^\circ \, t$

$= U\sin\theta - g\cos 30^\circ \cdot \dfrac{2U\sin\theta}{g\cos 30^\circ}$

$= -U\sin\theta$

$\because \tan\theta = \dfrac{\sqrt{3}}{2}$

$\therefore \sin\theta = \dfrac{\sqrt{3}}{\sqrt{7}} = \dfrac{\sqrt{21}}{7}$

$\therefore \cancel{V = \sqrt{21}} \quad V = -\dfrac{\sqrt{21}}{7}U$

$\therefore \text{speed} = \dfrac{\sqrt{21}}{7}U \text{ ms}^{-1}$

Answer $\dfrac{\sqrt{21}}{7}U \text{ ms}^{-1}$

COMMENTARY

This student used a very well presented and logical approach to obtain the required results. The working was easy for the examiner to follow and showed that the student had an appreciation of the ideas that were needed to obtain both results.

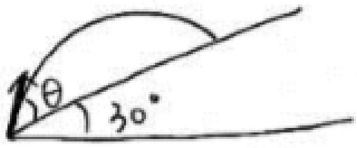
MARKS AWARDED: The student was awarded 11 marks out of a possible 11.

STUDENT B

RESPONSE

6 (a) Show that

$$\tan\theta = \frac{\sqrt{3}}{2}$$



[7 marks]

$v = \cos\theta \cdot u i + \sin\theta\, u j$

$a = -\cos 60^\circ a i \cancel{+} - \sin 60^\circ a j$

$\cancel{v_i = u_0 + at = 0}$

$\cancel{\cos\theta u - \cos 60^\circ a t = 0}$

$\cancel{t = \frac{\cos\theta u}{\cos 60^\circ} \quad t_{total} = \frac{2\cos\theta u}{\cos 60^\circ}}$

$v_j = \sin\theta u - \sin 60^\circ a t = 0.$

$t = \frac{\sin\theta u}{\sin 60^\circ a}$

Time when the ball hit the ground $\frac{2\sin\theta u}{\sin 60^\circ a}$.

$v_i = \cos\theta \cdot u - \cos 60^\circ a \cdot \frac{2\sin\theta u}{\sin 60^\circ a} = 0.$

$\cos\theta \cdot u = \frac{2\cos 60^\circ \cdot \sin\theta u}{\sin 60^\circ}$

$\frac{\sin\theta}{\cos\theta} = \frac{\sin 60^\circ}{2\cos 60^\circ} = \frac{\sqrt{3}}{2}$

6 (b) Find, in terms of *U*, the speed at which the ball first hits the plane.

[4 marks]

$v_i = \cos\theta \cdot u = \frac{2}{\sqrt{7}} u$.

$v_j \cancel{= u_0 + at} = \frac{4}{7} u$

$\cancel{= \sin\theta}$

Answer $\frac{4}{7} u$

COMMENTARY

In part (a), the student provided a good solution, but did not write $\tan\theta$ as part of their final line of working. In this case the examiner used the benefit of the doubt principle and awarded all of the marks, but the student did run the risk of not gaining the final mark by this omission.

In contrast, in part (b), the student was not able to identify a strategy and seemed to assume that the component perpendicular to the plane on impact would be the same as the component at *O*. This is probably an error made by comparing the motion to that above a horizontal plane.

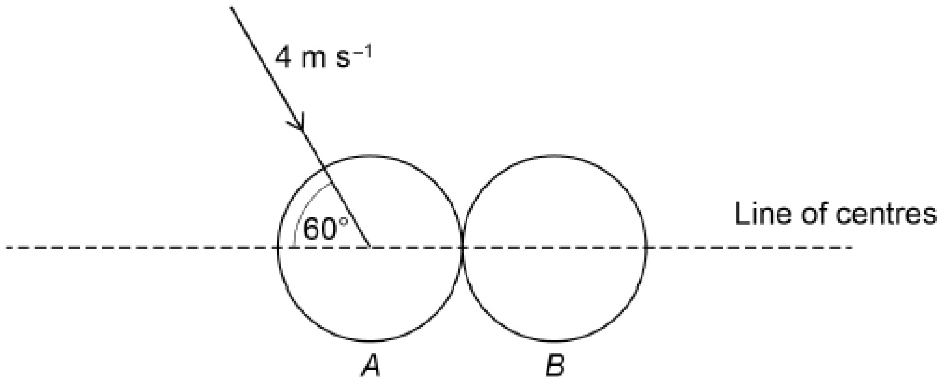
MARKS AWARDED: The student was awarded 7 marks out of a possible 11.

QUESTION 7

7 Two smooth spheres, *A* and *B*, are the same size.

Sphere *B* is initially at rest on a smooth horizontal surface.

Sphere *A* is moving at 4 m s^{-1} at an angle of 60° to the line of centres when it collides with *B*, as shown in the diagram.



The mass of *A* is 3 kg and the mass of *B* is 2 kg.

The coefficient of restitution between the spheres is 0.6

7 (a) Describe the direction in which *B* moves after the collision.

[1 mark]

7 (b) Find the speed of *B* after the collision.

[6 marks]

7 (c) Find the magnitude and direction of the velocity of *A* after the collision.

[4 marks]

7 (d) Find the magnitude of the impulse on *A* during the collision.

[2 marks]

MARK SCHEME

Q	Answer	Mark	Comments
7 (a)	*B* will move along the line of centres.	B1 **1**	B1: Correct statement about the line of centres.
7 (b)	Conservation of momentum along line of centres: $3 \times 4\cos 60° = 2v_B + 3v_A$ $6 = 2v_B + 3v_A$ Use of law of restitution: $v_A - v_B = -0.6(4\cos 60° - 0)$ $v_A - v_B = -1.2$ $v_A = v_B - 1.2$ $6 = 2v_B + 3(v_B - 1.2)$ $v_B = \frac{9.6}{5} = 1.9\text{ m s}^{-1}$	M1 A1 M1 A1 M1 A1 **6**	M1: Three term equation for conservation of momentum. Allow trig errors. A1: Correct equation. M1: Restitution equation. Allow sign / trig errors. A1: Correct equation. M1: Solving their equations. A1: Correct speed to 2 sf. Accept 1.92
7 (c)	Velocity along line of centres: $= 1.92 - 1.2 = 0.72$ Velocity perpendicular to line of centres: $= 4\sin 60 = 2\sqrt{3}$ Magnitude of velocity: $\sqrt{0.72^2 + (2\sqrt{3})^2} = 3.5\text{ m s}^{-1}$ Direction θ to line of centres: $\theta = \tan^{-1}\left(\frac{2\sqrt{3}}{0.72}\right) = 78°$	M1 M1 A1 A1 **4**	M1: Finding velocity along line of centres. M1: Finding velocity perpendicular to the line of centres. A1: Correct magnitude of velocity. AWRT 3.5 A1: Correct direction. AWRT 78
7 (d)	$I = 2 \times 1.92 = 3.8\text{ N s}$	M1A1F **2**	M1: Impulse equation with correct values and any signs. A1F: Correct impulse. AWRT 3.8 FT their velocity.
	Total	**13**	

STUDENT A

RESPONSE

7 (a) Describe the direction in which *B* moves after the collision.

[1 mark]

right ~~to~~ , parrallel to line of centres.

7 (b) Find the speed of *B* after the collision.

[6 marks]

$3 \times 4\cos 60 = \cancel{2V_B\cos\theta}.\ 3V_A + 2V_B.$

$\cancel{3 = V_B\cos\theta}.\quad 6 = 3V_A + 2V_B$

$4\cos 60 \times 0.6 = V_B - V_A.$

$1.2 = V_B - V_A.$

$2.4 = 2V_B - 2V_A$

$5V_A = 3.6$

$V_A = 0.72\ \text{m/s}.$

$V_B = 1.2 + 0.72 = 1.92\ \text{m/s}.$

Answer 1.92. m s^{-1}

7 (c) Find the magnitude and direction of the velocity of *A* after the collision.

[4 marks]

$V^2 = (4\sin 60)^2 + 0.72^2 = 12.51$

$V = 3.54 m/s$

$\tan\theta = \frac{4\sin 60}{0.72}$ $\theta = 78.3°$

Magnitude 3.54. m s^{-1}

Direction 78.3° to downwards.

7 (d) Find the magnitude of the impulse on *A* during the collision.

[2 marks]

$P = m\Delta V = 3\times(4\cos 60 - 0.72)$

$= 3.84$ Ns

Answer 3.84 N s

COMMENTARY

In part (a), the student was awarded the mark, although "along the line of centres" would have been a better answer.

In part (b), the student carefully sets up and solves the appropriate equations.

In part (c), the student uses both components to find both the magnitude and direction as required.

In part (d), the student uses the components parallel to the line of centres to find the correct impulse. The student did not realise that it would have been easier to find the magnitude of the impulse on *B*, which would be the same as the magnitude of the impulse on *A*.

MARKS AWARDED: The student was awarded 13 marks out of a possible 13.

STUDENT B

RESPONSE

7 (a) Describe the direction in which *B* moves after the collision.

[1 mark]

B will move ~~downwards and~~ to the right.

7 (b) Find the speed of *B* after the collision.

[6 marks]

$\because$ The velocity ~~along~~ perpendicular to the line of centres is unchanged.

$\therefore \begin{cases} m_A u_A + m_B u_B = m_A V_A + m_B V_B \\ \dfrac{V_B - V_A}{u_A - u_B} = 0.6 \end{cases}$

$\therefore \begin{cases} 3 \times 4\cos 60^\circ = 3V_A + 2V_B \\ V_B - V_A = 4\cos 60^\circ \times 0.6 \end{cases}$

$\therefore \begin{cases} 3V_A + 2V_B = 6 \\ V_B - V_A = ~~3.6~~\ 1.2 \end{cases}$

$\therefore V_A = 0.72$, ~~$V_B = \frac{84}{25} = 3.36$~~ $V_B = 1.92$

Answer ~~84/25~~ ~~3.36~~ 1.92 m s^{-1}

7 (c) Find the magnitude and direction of the velocity of *A* after the collision.

[4 marks]

$\because\ V = \sqrt{(4\sin 60^\circ)^2 + V_A^2}$

$= \sqrt{(4\sin 60^\circ)^2 + 0.72^2} = 3.54\ ms^{-1}$

$\cancel{= 3.47\ ms^{-1}}$

$\theta = \tan^{-1}\left(\frac{4\sin 60^\circ}{0.72}\right) = \cancel{86.2}\ 78.3^\circ$ / $\begin{pmatrix} 0.72 \\ -4\sin 60^\circ \end{pmatrix}$

Magnitude $\cancel{3.47}$ 3.54 m s^{-1}

$\begin{pmatrix} 0.72 \\ -4\sin 60^\circ \end{pmatrix} \longrightarrow$

Direction 78.3 ~~86.2~~ ° between the ~~horizon~~ line of centres. ~~downwards to the left~~ ~~right~~

7 (d) Find the magnitude of the impulse on *A* during the collision.

[2 marks]

$I = |m\Delta v| = \left|3 \times (\cancel{3.47}\ 3.54 - 4)\right| = \cancel{1.59}\ 1.38\ Ns$

Answer $\cancel{1.59}$ 1.38 N s

COMMENTARY

In part (a), the student gives a very common answer, which is not precise enough to gain the mark. To gain the mark here the phrase "the line of centres" needed to be stated.

The student provides good solutions to parts (b) and (c), showing a familiarity with the topic.

In part (d), the velocities are used to find the impulse rather than the components of the velocities. This was a common error. Impulse does seem to have been found to be a difficult concept by many of the students taking this examination.

Note that the final answers given by this student were not given to two significant figures as instructed.

MARKS AWARDED: The student was awarded 10 marks out of a possible 13.

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