OXFORD
INTERNATIONAL
AQA EXAMINATIONS

INTERNATIONAL A-LEVEL
MATHEMATICS

(9660)

Example responses with commentary: MA05, Unit M2

For teaching from September 2017 onwards

INTRODUCTION

This guide includes student responses to questions from the June 2019 International A-level Mathematics (9660) Paper 5, Unit M2.

The questions are presented with the mark schemes, student responses and commentaries from the Lead Examiner.

ASSESSMENT OBJECTIVES

The exams will measure how students have achieved the following Assessment Objectives:

- AO1: recall and select knowledge of mathematical facts, concepts, models and techniques required to solve problems in a variety of contexts.
- AO2: construct rigorous mathematical arguments and proofs through use of precise statements, mathematical manipulation, logical deduction, modelling assumptions and justifications to solve structured and unstructured problems, and to deduce, interpret and communicate results.

KEY TO MARK SCHEME ABBREVIATIONS

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
−x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

EXAMPLE RESPONSES

QUESTION 1

1 A boat moves so that its position, $\mathbf{r}$ metres, at time t seconds is given by

$$\mathbf{r} = (4e^{-0.5t} - 4)\,\mathbf{i} + (t + \sin t)\,\mathbf{j}$$

where the unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed east and north respectively.

1 (a) Find an expression for the velocity of the boat at time t.

[3 marks]

1 (b) Hence find the speed of the boat when $t = 5$

[2 marks]

1 (c) Find the magnitude of the acceleration of the boat when $t = 5$

[2 marks]

MARK SCHEME

Q	Answer	Mark	Comments
1(a)	$v = \frac{dr}{dt}$	M1	Attempt at differentiating either of the two components
	$v = -2e^{-0.5t}\mathbf{i} + (1 + \cos t)\,\mathbf{j}$	A1 A1	Correct $\mathbf{i}$ component Correct $\mathbf{j}$ component
1(b)	$v = -2e^{-0.5\times5}\mathbf{i} + (1 + \cos(5))\,\mathbf{j}$	M1	oe, e.g. column vector FT their answer to **(a)**
	$\lvert v\rvert = 1.29$ [m s^{-1}]	A1	AWRT 1.3 m s^{-1} from below
1(c)	$a = e^{-0.5t}\,\mathbf{i} - \sin t\,\mathbf{j}$	M1	FT their answer to part **(a)** in the form $v = ae^{-0.5t}\mathbf{i} + (b \pm \cos t)\,\mathbf{j}$ where $a \neq 0$
	$\lvert a\rvert = 0.962\,[\text{m s}^{-2}]$	A1	CAO Condone 0.96 [m s^{-2}] but not 0.963 [m s^{-2}]
	Total	**7**	

STUDENT A

RESPONSE

1 A boat moves so that its position, **r** metres, at time t seconds is given by

$$\mathbf{r} = (4e^{-0.5t} - 4)\,\mathbf{i} + (t + \sin t)\,\mathbf{j}$$

where the unit vectors **i** and **j** are directed east and north respectively.

1 (a) Find an expression for the velocity of the boat at time t.

[3 marks]

$v = \frac{dr}{dt} = (-2e^{-0.5t})i + (1 + \cos t)j$

Answer $(-2e^{-0.5t})i + (1 + \cos t)j$

1 (b) Hence find the speed of the boat when $t = 5$

[2 marks]

$(-2e^{-0.5(5)})i + (1 + \cos 5)j$ radians.

$\sqrt{(-2e^{-0.5\times5})^2 + (1+\cos 5)^2} = 1.29\text{ ms}^{-1}$

Answer 1.29 ms^{-1}

1 (c) Find the magnitude of the acceleration of the boat when $t = 5$

[2 marks]

$a = \frac{dv}{dt} = (e^{-0.5t})i + (-\sin t)j$

$\sqrt{(e^{-0.5\times5})^2 + (-\sin 5)^2} = 0.962\text{ ms}^{-2}$

Answer 0.962 ms^{-2}

COMMENTARY

The student has provided a perfect solution to the questions, showing all intermediate steps, clear methods and units with their answers, as well as making sure to use the radian mode on their calculator.

MARKS AWARDED: 7

STUDENT B

RESPONSE

1 A boat moves so that its position, $\mathbf{r}$ metres, at time t seconds is given by

$$\mathbf{r} = (4e^{-0.5t} - 4)\,\mathbf{i} + (t + \sin t)\,\mathbf{j}$$

where the unit vectors **i** and **j** are directed east and north respectively.

1 (a) Find an expression for the velocity of the boat at time t.

[3 marks]

$v = \frac{dr}{dt} = 4 \times (-\frac{1}{2}) e^{-0.5t}\, i + (1 + \cos t)\, j$

$= -2e^{-0.5t}\, i + (1 + \cos t)\, j$

Answer $v = -2e^{-0.5t}\, i + (1 + \cos t)\, j$

1 (b) Hence find the speed of the boat when $t = 5$

[2 marks]

when $t = 5$

$v = -2e^{-0.5 \times 5}\, i + (1 + \cos 5)\, j$

$= -2e^{-2.5}\, i + (1 + \cos 5)\, j$

Speed $= \sqrt{(-2e^{-2.5})^2 + (1 + \cos 5)^2}$

$= 2\ ms^{-1}$

Answer $2\ ms^{-1}$

1 (c) Find the magnitude of the acceleration of the boat when $t = 5$

[2 marks]

$a = \frac{dv}{dt}$ ~~$= (-2) \times (-2.5) e^{-2.5}$~~

$= (-2) \times (-0.5) e^{-0.5t}\, i + (1 - \sin t)\, j$

$= e^{-0.5t}\, i + (1 - \sin t)\, j$

when $t = 5$ $a = e^{-2.5}\, i + (1 - \sin 5)\, j$

$a = \sqrt{(e^{-2.5})^2 + (1 - \sin 5)^2}$

$= 0.917\ ms^{-2}$

Answer $0.917\ ms^{-2}$

COMMENTARY

The student has provided a good solution to part (a), achieving M1 A1 A1. However, they do not use radians for the calculation in part (b) and so are awarded M1 A0. In part (c) they do not correctly differentiate their velocity vector, meaning that they are awarded M0 for this part.

MARKS AWARDED: 4

QUESTION 2

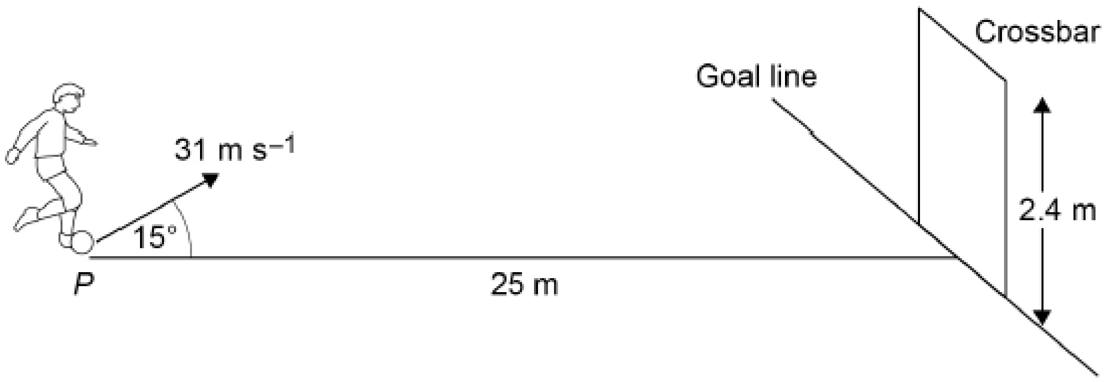
2 A footballer practises on horizontal ground by kicking a ball from a point *P* directly towards a goal.

The point *P* is such that

it is a perpendicular distance of 25 metres from the goal line

it is directly in front of the centre of the goal.

The ball leaves the footballer's foot with a speed of $31\ \text{m s}^{-1}$ at an angle of 15° to the horizontal, as shown in the diagram below.



The ball may be modelled as a particle.

2 (a) Show that the time the ball takes to move the horizontal distance of 25 metres is 0.83 seconds, correct to two significant figures.

[1 mark]

2 (b) To score a goal the ball must pass under the crossbar. The crossbar of the goal is 2.4 metres above the ground.

Determine whether or not the footballer scores a goal with this kick.

[4 marks]

MARK SCHEME

Q	Answer	Mark	Comments
2(a)	$31\cos(15) = 29.9[437 \text{ m s}^{-1}]$ $\frac{25}{31\cos(15)} = 0.834[9 \text{ s}]$	**B1**	Must show working, such as $\frac{25}{29.9...}$, $\frac{25}{31\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)}$, $\frac{25\left(\sqrt{6}-\sqrt{2}\right)}{31}$ or gives the time to at least 3 significant figures.
2(b)	$31\sin(15)$ or $8.0[234 \text{ m s}^{-1}]$	**M1**	PI or seen in any working
	$s = 8.0234 \times 0.8349 - 0.5 \times 9.8 \times 0.8349^2$	**m1**	Use of $s = ut + ½at^2$ and $a = \pm 9.8$ with $t = 0.83$ or better
	$s = 3.28[31 \text{ m}]$	**A1**	AWRT 3.3 m The answer must come from considering the height of ball at the goal line and not from a calculation of the maximum height reached by the ball during its flight.
	No goal is scored as $3.28 > 2.4$	**E1F**	Must have scored at least **M1 m0** Must compare their 3.28 (or 3.3) with 2.4 or state '0.88.. m over' and give correct conclusion.
	Total	**5**	

STUDENT A

RESPONSE

2 (a) Show that the time the ball takes to move the horizontal distance of 25 metres is 0.83 seconds, correct to two significant figures.

[1 mark]

$\dot{x} = V\cos\theta$

$31\cos 15 = \dot{x}$ = ~~Veloci~~ Horizontal velocity. $= 29.9\,ms^{-1}$

$V = \frac{s}{t}$ $\quad 29.9 = \frac{25}{t}$

$\frac{25}{29.9} = t = 0.8349s \approx 0.83s$ 2.s.f.

2 (b) To score a goal the ball must pass under the crossbar. The crossbar of the goal is 2.4 metres above the ground.

Determine whether or not the footballer scores a goal with this kick.

[4 marks]

$s = ut + \frac{1}{2}at^2$

$y = vt\sin\theta - \frac{1}{2}gt^2$

$t = 0.83s$

$y = 31 \times \sin 15 \times 0.83 - 4.9 \times 0.83^2 = 3.28m$ at the goal line

$3.28 - 2.4 = 0.88m$

He wil not score a goal. 0.88m above cross bar.

COMMENTARY

This student has provided a correct solution to both part (a) and part (b). In part (a) they show the time to four significant figures, which satisfies the condition of showing at least one more significant figure than the value given in the question. In part (b) they calculate correctly the height of the ball at the goal line, compare this height with that of the crossbar and then give the correct conclusion that, as the ball is 0.88m above the crossbar, no goal is scored.

MARKS AWARDED: 5

STUDENT B

RESPONSE

2 (a) Show that the time the ball takes to move the horizontal distance of 25 metres is 0.83 seconds, correct to two significant figures.

[1 mark]

$s = vt$

$25 = 31\cdot\cos 15\cdot t$

$0.83 = t$

2 (b) To score a goal the ball must pass under the crossbar. The crossbar of the goal is 2.4 metres above the ground.

Determine whether or not the footballer scores a goal with this kick.

[4 marks]

$v = v_0 + at$

$v = 31\cdot\sin(15) + (-9.8)\cdot 0.83$

$v = -0.11$

$v^2 = v_0^2 + 2as$

$(-0.11)^2 = (31\sin(15))^2 + 2\cdot(-9.8)\cdot s$

$-64.4 = -19.6s$

$3.3 = s$

He can't cross the crossbar.

COMMENTARY

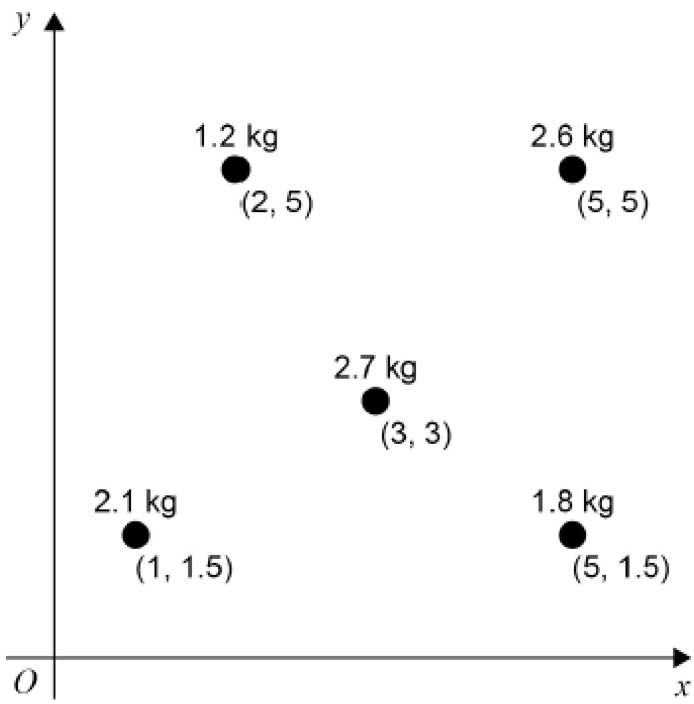
This student has not shown sufficient working in part (a) to be awarded the mark, as they do not show an intermediate step where the time t is the subject of the equation, nor do they give the time to more significant figures than that shown in the question. In part (b), they find that the height of the ball at the goal line is 3.3 [metres], but they do not interpret this value in terms of whether or not a goal is scored, meaning that they are not awarded the E1 mark.

MARKS AWARDED: 3

QUESTION 4

4 A system of five particles, along with their masses and coordinates, is shown in **Figure 1**.

Figure 1

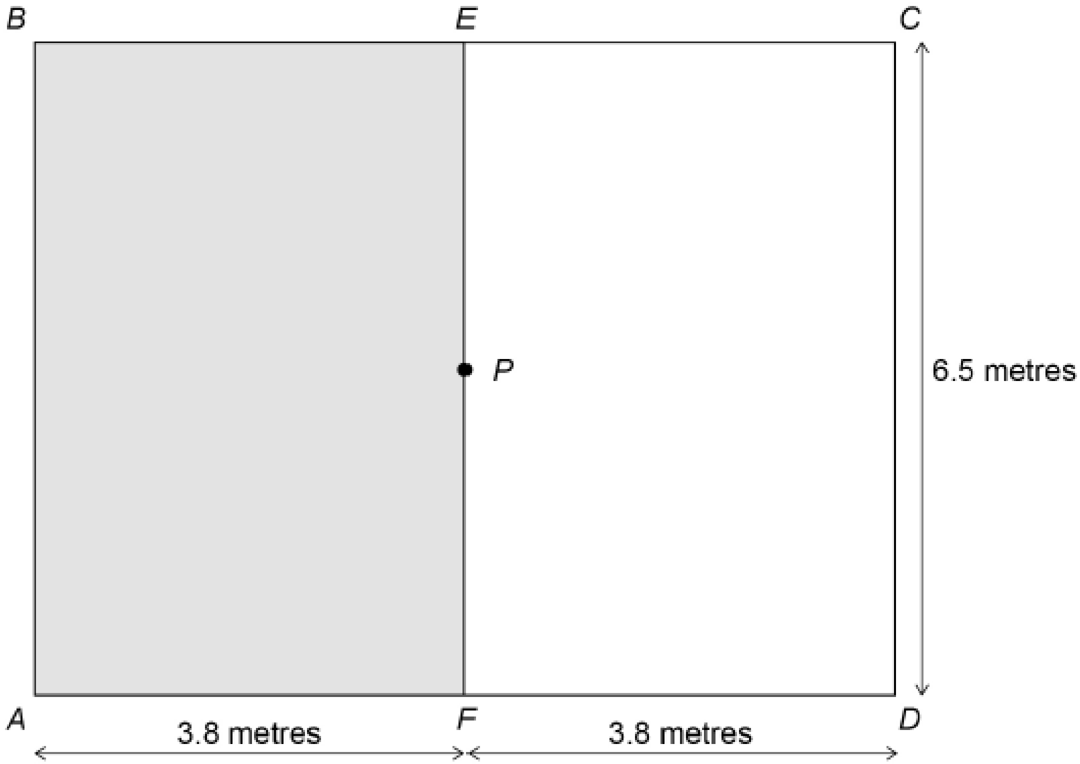


4 (a) Find the coordinates of the centre of mass of the system of particles.

[4 marks]

4 (b) The rectangular board *ABCD* is made by joining together the two uniform rectangular boards *ABEF* and *ECDF* with dimensions as shown in **Figure 2**.

Figure 2



The point *P* is the midpoint of the line *EF*.

The board *ABEF* has mass $1.5m$ kilograms.

The board *ECDF* has mass m kilograms.

The board *ABCD* is freely suspended from the point *B* and is in equilibrium.

Find, to the nearest degree, the angle between *BP* and the vertical.

[7 marks]

MARK SCHEME

Q	Answer	Mark	Comments
4(a)	$\bar{X} = \frac{(2.1\times1)+(1.2\times2)+(2.7\times3)+(1.8\times5)+(2.6\times5)}{10.4}$	M1	M1 for at least 3 multiplications and additions
	$\bar{X} = \frac{173}{52}$	A1	oe. AWRT 3.3
	$\bar{Y} = \frac{(2.1\times1.5)+(1.2\times5)+(2.7\times3)+(1.8\times1.5)+(2.6\times5)}{10.4}$	M1	M1 for at least 3 multiplications and additions
	$\bar{Y} = \frac{659}{208}$	A1	oe. AWRT 3.2
4(b)	$\bar{X} = \frac{1.5m\times1.9+m\times5.7}{2.5m}$	M1	PI, allow one slip
	$\bar{X} = \frac{171}{50}$	A1	oe (may be seen on diagram)
	$\bar{Y} = \frac{13}{4}$	B1	oe (may be seen on diagram)
	$\tan^{-1}(3.25/3.8) = 40.5[39\ldots]^\circ$	B1	or $\tan^{-1}(3.8/3.25) = 49.4[608\ldots]^\circ$
	$\tan^{-1}(3.25/3.42) = 43.5[40\ldots]^\circ$	B1	or $\tan^{-1}(3.42/3.25) = 46.4[599\ldots]^\circ$
	$43.5[40\ldots] - 40.5[39\ldots]$	m1	PI by correct final answer oe, such as $49.4[608\ldots] - 46.4[599\ldots]$
	$= 3^\circ$	A1	CAO
	Total	**11**	

ALT			
4(b)	$\bar{X} = \frac{1.5m\times1.9+m\times5.7}{2.5m}$	M1	PI, allow one slip
	$\bar{X} = \frac{171}{50}$	A1	oe (may be seen on diagram)
	$\bar{Y} = \frac{13}{4}$	B1	oe (may be seen on diagram)
	[Length *BP* =] 5.00025 [m]	B1	
	[Length *B* to COM =] 4.71793 [m]	B1	
	$\cos\theta = \frac{5.00025^2 + 4.71793^2 - 0.38^2}{2\times5.00025\times4.71783}$	m1	PI by correct final answer
	$[\theta] = 3^\circ$	A1	CAO

STUDENT A

RESPONSE

4 (a) Find the coordinates of the centre of mass of the system of particles.

[4 marks]

$1.2 \times 2 + 2.6 \times 5 + 2.7 \times 3 + 1.8 \times 5 + 2.1 \times 1 =$

$(1.2 + 2.6 + 2.7 + 1.8 + 2.1)\bar{x}$

$\frac{173}{5} = \frac{52}{5}\bar{x}$

$3.3269\ldots = \bar{x}$

$1.2 \times 5 + 2.6 \times 5 + 2.7 \times 3 + 1.8 \times 1.5 + 2.1 \times 1.5 =$

$(1.2 + 2.6 + 2.7 + 1.8 + 2.1)\bar{y}$

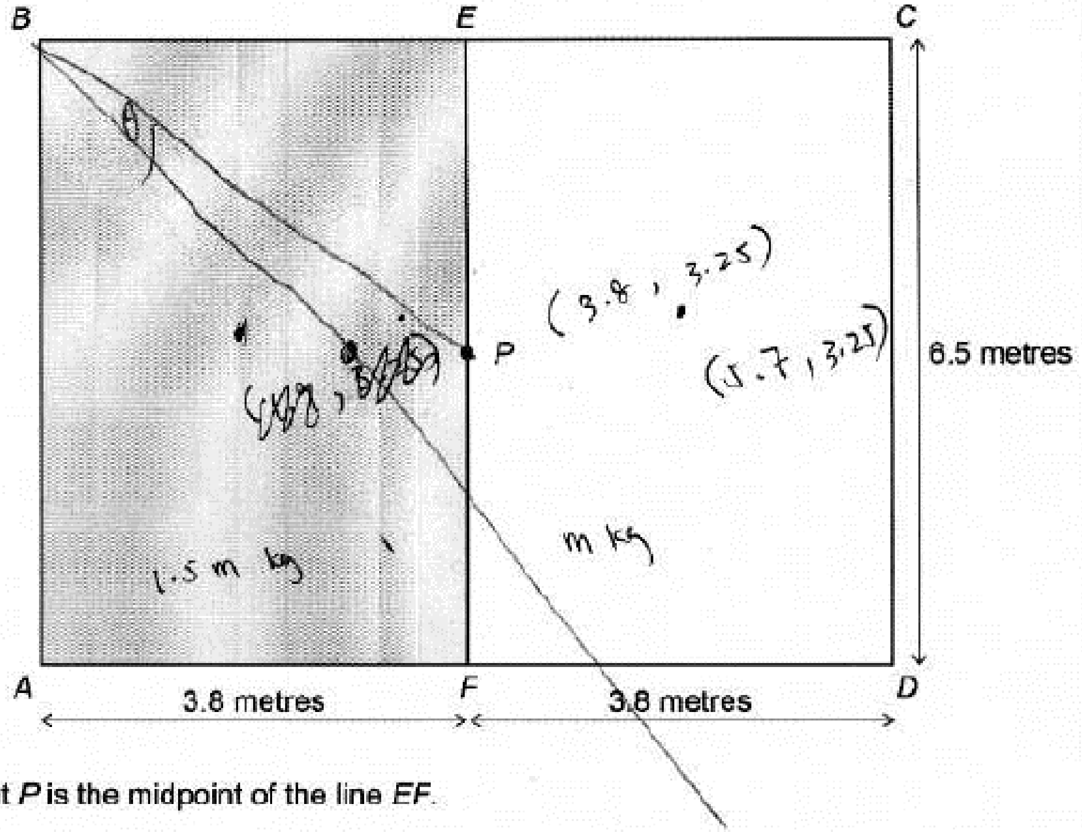
$\frac{659}{20} = \frac{52}{5}\bar{y}$

$3.1683\ldots = \bar{y}$

Answer $(3.33, 3.17)$

4 (b) The rectangular board *ABCD* is made by joining together the two uniform rectangular boards *ABEF* and *ECDF* with dimensions as shown in **Figure 2**.

Figure 2



The point *P* is the midpoint of the line *EF*.

The board *ABEF* has mass $1.5m$ kilograms.

The board *ECDF* has mass m kilograms.

The board *ABCD* is freely suspended from the point *B* and is in equilibrium.

Find, to the nearest degree, the angle between *BP* and the vertical.

[7 marks]

center of mass ABCD

$1.9 \times 1.5m + 5.7 \times m = (1.5m + m)\bar{X}$

$2.85m + 5.7m = 2.5m\bar{X}$

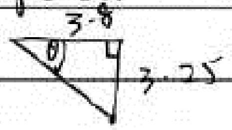
$3.42 = \bar{X}$

$1.5m + m$

$3.25 \times 1.5m + 3.25 \times m = (\quad)\bar{Y}$

$3.25 = \bar{Y}$

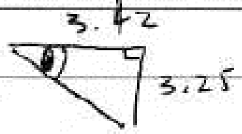
angle between BP and BC :



$\tan\theta = \frac{3.25}{3.8}$

$\theta = 40.5392...^\circ$

angle between vertical and BC :



$\tan\theta = \frac{3.25}{3.42}$

$\theta = 43.5400...^\circ$

$43.5400 - 40.5392 = 3.0008...^\circ$

Answer 3°

COMMENTARY

This student's answer is awarded all of the marks available for parts (a) and (b). In part (a), they show their full method and give answers as fractions, before giving rounded decimals as their final answer. In part (b) they make full use of the diagram given in the question to make annotations and to establish what angle question is asking for. They then proceed via the main method shown in the mark scheme to arrive at the correct answer.

MARKS AWARDED: 11

STUDENT B

RESPONSE

4 (a) Find the coordinates of the centre of mass of the system of particles.

[4 marks]

m	1.2	2.6	2.7	2.1	1.8	10.4
x	2	5	3	1	5	
mx	2.4	13	8.1	2.1	9	34.6

$\bar{x} = \frac{34.6}{10.4} = 3.33$

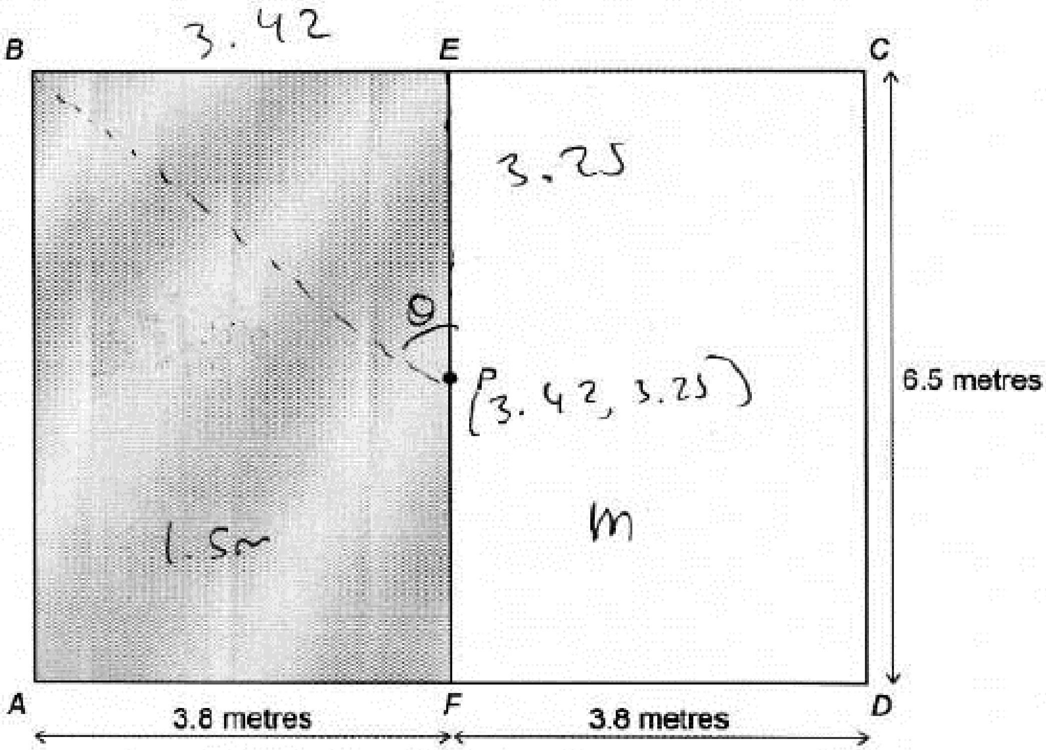
m	1.2	2.6	2.7	2.1	1.8	10.4
y	5	5	3	1.5	1.5	
my	6	13	8.1	3.15	2.7	32.95

$\bar{y} = \frac{32.95}{10.4} = 3.17$

Answer (3.33, 3.17)

4 (b) The rectangular board *ABCD* is made by joining together the two uniform rectangular boards *ABEF* and *ECDF* with dimensions as shown in **Figure 2**.

Figure 2



The point *P* is the midpoint of the line *EF*.

The board *ABEF* has mass 1.5*m* kilograms.

The board *ECDF* has mass *m* kilograms.

The board *ABCD* is freely suspended from the point *B* and is in equilibrium.

Find, to the nearest degree, the angle between *BP* and the vertical.

[7 marks]

m	1.5m	m	2.5m		
x	1.9	5.7		$\bar{x}$	$\frac{8.55m}{2.5m} = 3.42$
mx	2.85m	5.7m	8.55m		

$\bar{y} = 3.25$ centre of mass $(3.42, 3.25)$

$\tan\theta = \frac{3.42}{3.25}$

$\theta = 46.46 = 46°$

COMMENTARY

This student's answer is awarded all of the marks available for part (a), but only 4 marks for part (b). In part (b) the student is awarded M1 A1 B1 B0 B1 m0 A0, as they correctly find the coordinates for the centre of mass of the composite rectangular board, as well as finding one of the two correct angles needed to reach the solution.

MARKS AWARDED: 8

QUESTION 5

5 A child of mass 35 kg starts from rest at the top of a slide.

The slide is inclined at 25° to the horizontal.

The coefficient of dynamic friction between the child and the slide is 0.2

The child may be modelled as a particle.

5 (a) Draw a diagram to show all the forces acting on the child, writing down the names of the forces on your diagram.

[1 mark]

5 (b) (i) Find the acceleration of the child down the slide.

[5 marks]

5 (b) (ii) Calculate the work done against friction when the child has moved through a vertical height of 2.2 metres.

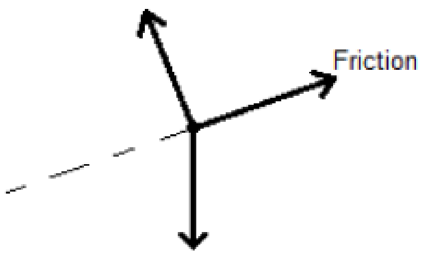
[2 marks]

5 (c) State how your answer to part **(b) (i)** would be different if the child was not modelled as a particle.

Explain your answer.

[2 marks]

MARK SCHEME

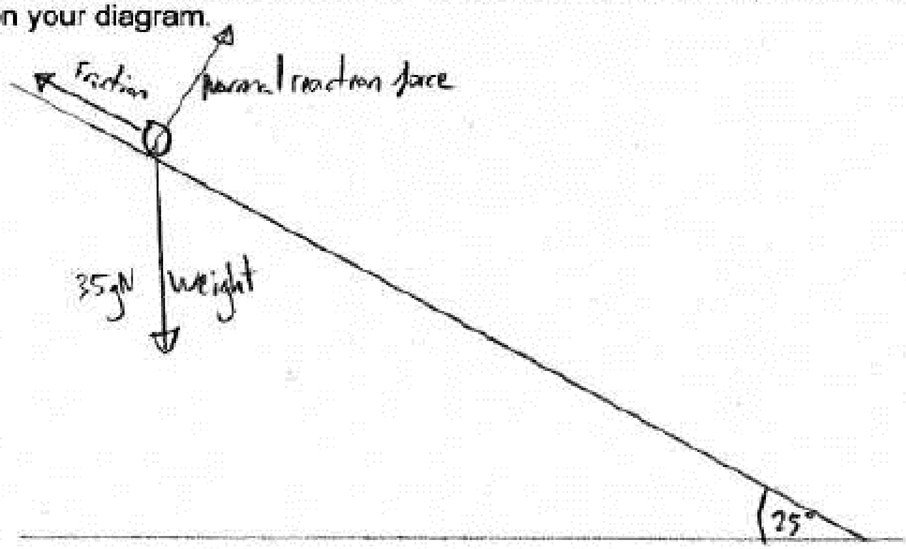
Q	Answer	Mark	Comments
5(a)		B1	Must have names on the three arrows, not symbols unless the symbols are defined. Do not accept 'gravity' in place of weight.
5(b)(i)	[Component of weight down the slope = 35 × 9.8 sin(25°)] = 144.9[58 N]	B1	oe, AWRT 145 N
	[Normal reaction force on child = 35 × 9.8 cos(25°)] = 310.8[64 N]	M1	oe, AWRT 311 N PI
	[Friction on child = 0.2 × 310.863…] = 62.1[73 N]	A1	oe
	Resultant force down the slope = 144.958 – 62.173 = 82.7[85 N]	M1	FT their force up and down the slope
	Resultant force = ma a = 82.785 / 35 = 2.37 [m s^{-2}]	A1	CAO, ISW Allow 2.4 m s^{-2} but not 2.36 m s^{-2}
5(b)(ii)	[Distance covered down the slope = 2.2 / sin(25°)] = 5.2[06 m]	M1	AWRT 5.2 metres PI
	[$W = Fd$ = 62.173 × 5.206] = 324 [J]	A1F	FT their friction force from **(b)(i)** with the correct distance Condone negative answers No ISW
5(c)	[Acceleration would be] less	E1	Not 'different' Not just 'air resistance'
	[Greater] air resistance would reduce the resultant force [acting on the child due to the child now having a larger surface area]	E1	Allow any plausible explanation related to the resultant force reducing or more work done against resistive forces.
	Total	**10**	

STUDENT A

RESPONSE

5 (a) Draw a diagram to show all the forces acting on the child, writing down the names of the forces on your diagram.

[1 mark]



5 (b) (i) Find the acceleration of the child down the slide.

[5 marks]

Component of weight down the slope:

$mg\sin\theta$: $35 \times 9.8 \times \sin 25 =$ 145 N

Friction:

$F = \mu R$ $\quad R = mg\cos\theta$ $\quad F = \mu mg\cos\theta$

$0.2 \times 35 \times 9.8 \times \cos 25 = 62.2N$

Resultant force:

$145N - 62.2N = 82.8N$

$F = ma$ $\quad \frac{82.8N}{35kg} = 2.37ms^{-2}$

$\frac{F}{m} = a$

Answer $2.37ms^{-2}$

5 (b) (ii) Calculate the work done against friction when the child has moved through a vertical height of 2.2 metres.

[2 marks]

Work done = Force x distance

distance in direction of force a:

$\sin 25 = \frac{2.2}{H}$ $H = \frac{2.2}{\sin 25}$ = 5.21m

5.21 x ~~m~~ 62.2N = 324J

Answer 324J

5 (c) State how your answer to part **(b) (i)** would be different if the child was not modelled as a particle.

Explain your answer.

[2 marks]

The Volume of the child would have to ~~as~~ accounted for and so the acceleration may be lower.

COMMENTARY

This student's response to part (a) includes a good diagram of the situation as well as correct names for each of the three forces acting on the child; in particular, the student writes 'weight' rather than the popular misconception 'gravity' for the gravitational force acting on the child. The student goes on to be awarded the full 7 marks across (b)(i) and (b)(ii). The response in part (c) gains one mark for the correct statement that the acceleration being lower, but the explanation is not complete enough for both marks.

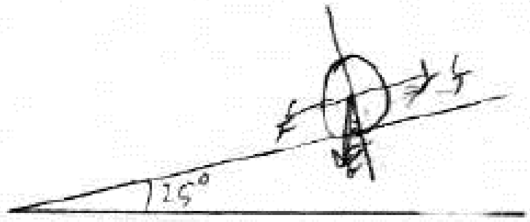
MARKS AWARDED: 9

STUDENT B

RESPONSE

5 (a) Draw a diagram to show all the forces acting on the child, writing down the names of the forces on your diagram.

[1 mark]



5 (b) (i) Find the acceleration of the child down the slide.

[5 marks]

$f = N\mu$

$f = 35 \cdot 9.8 \cdot \cos 25 \cdot 0.2$

$= 62.2N$

$F = ma$

$(mg\sin 25 - f) = ma$

$(35 \cdot 9.8 \cdot \sin 25 - 62.2) = 35a$

$a = 2.36$

5 (b) (ii) Calculate the work done against friction when the child has moved through a vertical height of 2.2 metres.

[2 marks]

$s = \frac{12.2}{\sin 25} = 28.9$

$w = Fs$

$= 62.2 \times 28.9\text{m}$

$= 1797.58\text{J}$

Answer ~~1797.58~~ J 1797.58 J

5 (c) State how your answer to part **(b) (i)** would be different if the child was not modelled as a particle.

Explain your answer.

[2 marks]

air force

COMMENTARY

In this student's answer to part (a), undefined labels are used whereas the question asked for names of the forces to be written down, and so the mark for part (a) is not awarded. The student is close to achieving all 5 marks in part (b)(i), but they round prematurely which results in an incorrect final answer. Therefore, the student is awarded 4 of the 5 marks, missing out on the final A1 mark. In part (b)(ii) the student uses an incorrect value in their calculations and so arrives at an incorrect final answer. The response in part (c) is not enough to be awarded either of the 2 marks available.

MARKS AWARDED: 4

QUESTION 8

8 A golf ball is hit from a point on horizontal ground with a speed u at a fixed angle θ to the ground.

Assuming air resistance is negligible, prove that the horizontal distance the golf ball travels before hitting the ground for the first time is proportional to u^2

[5 marks]

MARK SCHEME

Q	Answer	Mark	Comments
8	[vertical comp. of velocity =] $u \sin\theta$	B1	May be seen as part of a correct formula for 'vertical distance' involving t
	[$v = u + at$ At max. height $v = 0$] $t = \frac{u \sin\theta}{g}$	M1	Allow one slip
	[Time of flight =] $2 \times \frac{u \sin\theta}{g}$	A1	oe, for instance $\frac{u \sin\theta}{4.9}$
	[horizontal comp. of velocity =] $u \cos\theta$	B1	May be seen as part of a correct formula for 'horizontal distance' involving t
	[Range = $2 \times \frac{u \sin\theta}{g} \times u \cos\theta$ =] $\frac{2u^2 \sin\theta \cos\theta}{g}$	A1	ISW, but must get to a correct formula involving u^2 Allow 9.8 in place of g No errors seen
	Total	**5**	

STUDENT A

RESPONSE

8 A golf ball is hit from a point on horizontal ground with a speed u at a fixed angle θ to the ground.

Assuming air resistance is negligible, prove that the horizontal distance the golf ball travels before hitting the ground for the first time is proportional to u^2

$x \propto u^2$

[5 marks]

$s = ut + \frac{1}{2}at^2$

$0 = u\sin\theta\, t - \frac{1}{2} \times 9.8\, t^2$

$4.9t^2 = u\sin\theta\, t$

$t = \frac{u\sin\theta}{4.9}$

$x = v\cos\theta\, t$

$= u\cos\theta \left(\frac{u\sin\theta}{4.9}\right)$

$= \frac{u^2\cos\theta\sin\theta}{4.9}$

$x = ku^2$

COMMENTARY

In this response the student is awarded the full 5 marks. They set the vertical displacement to 0, which is only true at the start and end of this motion, and then find the time at which the golf ball first hits the ground after being hit. This time of flight is then used with the horizontal component of velocity to arrive correctly at an expression for the range in terms of u2, which was the required result.

MARKS AWARDED: 5

STUDENT B

RESPONSE

8 A golf ball is hit from a point on horizontal ground with a speed u at a fixed angle θ to the ground.

Assuming air resistance is negligible, prove that the horizontal distance the golf ball travels before hitting the ground for the first time is proportional to u^2

[5 marks]

$y = x\tan x - \frac{g}{2}\left(\frac{x}{v\cos x}\right)^2$ $\qquad s\uparrow = 0$

$0 = u\sin\theta t - 4.9t^2$

$y = ku^2$ $\qquad 4.9t^2 = u\sin t$

$t = \frac{u\sin\theta}{4.9}$

$u^2 = u^2\cos^2\theta$

$x = vt\cos\theta$

COMMENTARY

This student's response is awarded B1 M1 A1 M0 A0. The student uses correctly the vertical component of the initial velocity to formulate a correct equation to find the time of flight of the golf ball. They then proceed to find the correct expression for the time of flight. They do not, however, write down the horizontal component of velocity and so they are not awarded the final 2 marks.

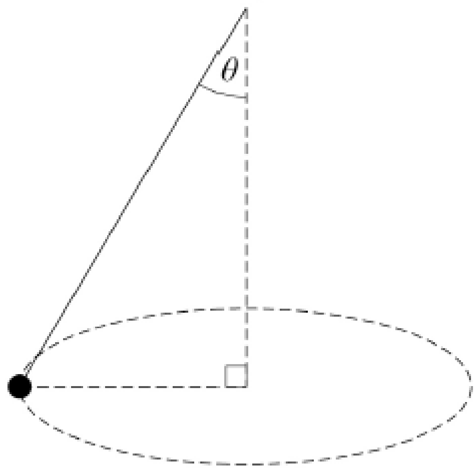
MARKS AWARDED: 3

QUESTION 9

9 A particle of mass 0.35 kg is attached to one end of a light inextensible string.

The other end of the string is attached to a fixed point.

The particle is set into circular motion so that the string remains taut and makes a fixed angle θ to the vertical, as shown in the diagram.



9 (a) Find, in terms of θ, the magnitude of the resultant force which acts on the particle.

[2 marks]

9 (b) Explain why the kinetic energy of the particle does not change even though there is a resultant force acting on the particle.

[2 marks]

9 (c) The radius of the horizontal circle followed by the particle is 0.62 metres, and the time for one revolution of the circle is 0.48 seconds.

Determine the angle θ.

[6 marks]

MARK SCHEME

Q	Answer	Mark	Comments
9(a)	Vertical component of tension, $T\cos\theta = 0.35g$ Horizontal component of tension, $T\sin\theta = F$ and eliminates T	M1	oe, for instance $T\cos\theta = 3.43$
	$F = 0.35g\tan\theta$	A1	oe, for instance $F = 3.43\tan\theta$ Condone answer left in terms of m, i.e. $F = mg\tan\theta$
9(b)	[The centripetal (or resultant)] force [on the particle acts] perpendicular [to the] velocity [of the particle]	E1	Condone 'speed'
	[The centripetal (or resultant) force does] no work [on the particle, so does not change the kinetic energy]	E1	
9(c)	Speed of particle $[v =]\ 2 \times \pi \times 0.62 / 0.48$	M1	Angular speed of particle $[\omega =]\ 2 \times \pi / 0.48$
	$[v =]$ 8.11[578 m s^{-1}] oe	A1	$[\omega =]$ 13.0[899… rad s^{-1}] oe
	$0.35g\tan\theta = 0.35v^2/r$ or $0.35g\tan\theta = 0.35\omega^2 r$ or [Resultant force =] 37.18… [N]	B1	oe accept 3.43 or mg for $0.35g$
	$\tan\theta = v^2/gr$ or $\tan\theta = \omega^2 r/g$	M1	Sight or attempt of isolating for $\tan\theta$
	$\tan\theta = 8.11578^2/(9.8 \times 0.62)$ oe or $\tan\theta = 13.0899\ldots^2 \times 0.62/9.8$ oe	m1	Only FT their speed, no other slips PI by correct answer
	84.7[°]	A1	CAO, AWRT 85[°] oe, for instance 1.48 [rad]
	Total	**10**	

STUDENT A

RESPONSE

9 (a) Find, in terms of θ, the magnitude of the resultant force which acts on the particle.

[2 marks]

Mg Mg

$mg = T\cos\theta$ $\frac{mg}{\cos\theta} = T$

Resultant force = Centripetal force.

$T\sin\theta$ = resultant force

$\frac{mg}{\cos\theta} \times \sin\theta$ = Resultant force. $Mg\tan\theta$ = resultant force.

$0.35 \times 9.8\tan\theta = 3.43\tan\theta$

Answer $3.43\tan\theta$

9 (b) Explain why the kinetic energy of the particle does not change even though there is a resultant force acting on the particle.

[2 marks]

The resultant force is always perpendicular to the direction of motion of the particle so there is no work done on the particle and so kinetic energy is constant. Speed remains constant as well.

9 (c) The radius of the horizontal circle followed by the particle is 0.62 metres, and the time for one revolution of the circle is 0.48 seconds.

Determine the angle θ.

[6 marks]

$F_c = m\omega^2 r$ $\omega = \frac{2\pi}{T}$ $\omega = \frac{2\pi}{0.48s} = 13.1\,\text{rad}\,s^{-1}$

$0.35 \times (13.1)^2 \times 0.62 = 37.182N \approx 37.2N$

$mg\tan\theta = 37.2N = 3.43\tan\theta$

$\frac{37.2}{3.43} = \tan\theta = 10.8$

$\theta = 84.7°$

COMMENTARY

This student's response is awarded all of the marks in each of the three parts of the question. In particular, the student gives a very good full explanation in part (b) and provides a very clear method for determining the angle in part (c).

MARKS AWARDED: 10

STUDENT B

RESPONSE

9 (a) Find, in terms of θ, the magnitude of the resultant force which acts on the particle.

[2 marks]

~~F = 0.35 × 9.8 ×~~

$F =$ ~~0.359~~ $0.35 \times 9.81 \times \tan\theta$

$= 3.43 \tan\theta$

Answer $F = 3.43 \tan\theta$

9 (b) Explain why the kinetic energy of the particle does not change even though there is a resultant force acting on the particle.

[2 marks]

because the ~~[illegible]~~ ω is not change

9 (c) The radius of the horizontal circle followed by the particle is 0.62 metres and the time for one revolution of the circle is 0.48 seconds.

Determine the angle θ.

2π rad = 0.48s

[6 marks]

2π rad = 0.48 s

$\omega = \frac{25}{6}\pi$ rads^{-1}

$F = m\omega^2 r$

$3.43\tan\theta = 0.35 \times (\frac{25}{6}\pi)^2 \times \frac{0.6^2}{\tan\theta}$

$3.43\tan^2\theta = 21.59$

$\tan\theta = 2.50886$

~~θ = 70.934~~ ~~68.268~~

θ = ~~21.73~~ $68.27°$

Answer ~~21.73°~~ $68.27°$

COMMENTARY

This student's response is awarded both marks available in part (a), as they arrive at the correct final answer. The response given in part (b) does not explain why the kinetic energy remains unchanged even though a resultant force acts on the particle, and so it is not awarded any marks. In part (c) the student is awarded M1 A1 B0 M0 M0, A1 as they find correctly the angular velocity of the particle, but they do not receive the B1 mark as they do not write down a correct equation for the resultant force. The M1 is not awarded at they end up with an equation for $\tan^2\theta$.

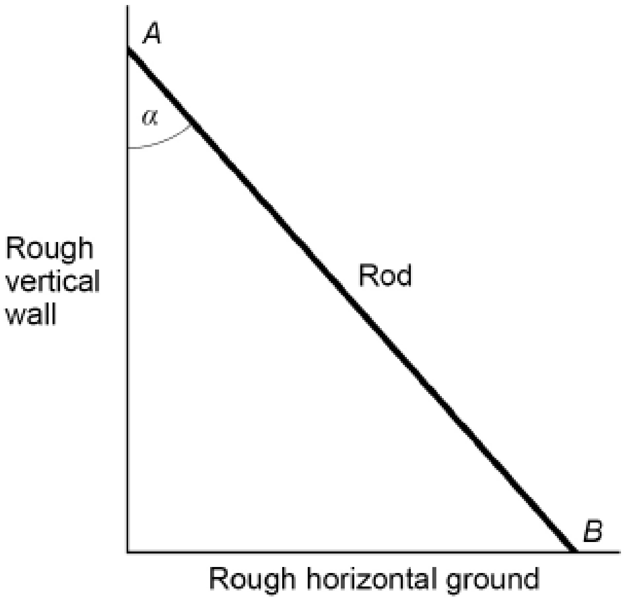
MARKS AWARDED: 4

QUESTION 10

10 A uniform rod, AB, of mass M is in equilibrium, with one end in contact with a rough vertical wall. The other end is on rough horizontal ground.

The coefficient of friction between the wall and the rod is μ and the coefficient of friction between the ground and the rod is also μ.

The rod makes an angle α with the wall, as shown in the diagram.



Given that the rod is on the point of slipping, find tan α in terms of μ.

[8 marks]

MARK SCHEME

Q	Answer	Mark	Comments
10	Taking moments about *A* or *B*	M1	Any attempt
	[Moments about *B*] $LMg \sin\alpha = 2LR_1 \cos\alpha + 2L\,\mu R_1 \sin\alpha$	A1	L = length of rod R_1 = normal reaction on rod from wall Condone cancelled L
	$\tan\alpha = \dfrac{2R_1}{Mg - 2\mu R_1}$	A1	
	Forces in equilibrium Horizontal: $\mu R_2 = R_1$	B1	R_2 = normal reaction on rod from ground Accept 'f_1' and 'f_2' in place of μR_1 and μR_2 respectively.
	Forces in equilibrium Vertical: $Mg = R_2 + \mu R_1$	B1	
	$\mu Mg = R_1 + \mu^2 R_1$ $R_1 = \dfrac{\mu Mg}{1+\mu^2}$	m1	Attempt at eliminating R_2
	$\tan\alpha = \dfrac{\dfrac{2\mu Mg}{1+\mu^2}}{Mg - 2\mu \times \dfrac{\mu Mg}{1+\mu^2}}$	m1	Substituting into correct relation for $\tan\alpha$
	$\tan\alpha = \dfrac{2\mu}{1-\mu^2}$	A1	CAO
	Total	**8**	

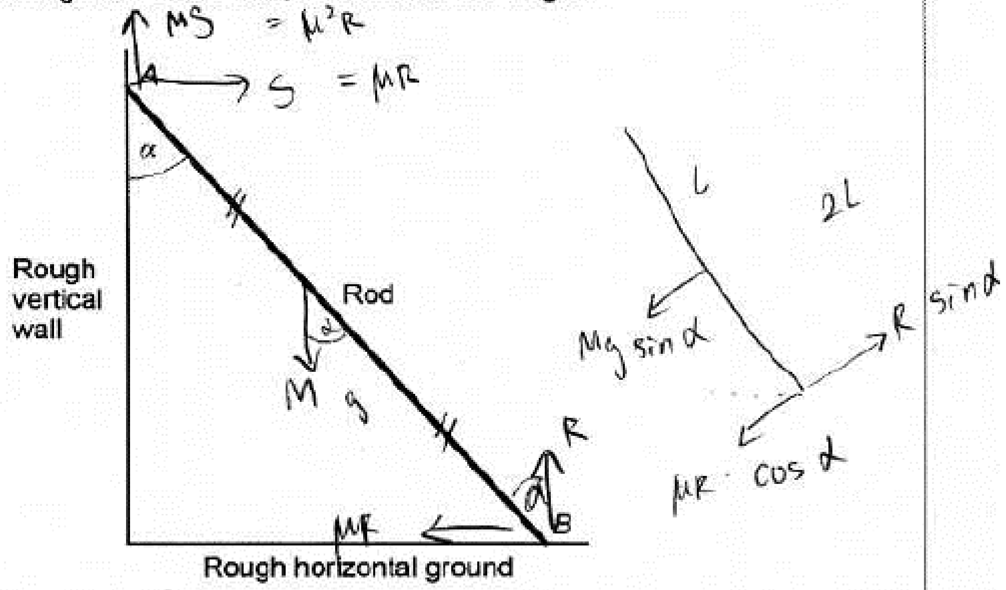
STUDENT A

RESPONSE

10 A uniform rod, AB, of mass M is in equilibrium, with one end in contact with a rough vertical wall. The other end is on ~~rough horizontal~~ ground.

The coefficient of friction between the wall and the rod is μ and the coefficient of friction between the ground and the rod is also μ.

The rod makes an angle α with the wall, as shown in the diagram.



Given that the rod is on the point of slipping, find $\tan\alpha$ in terms of μ.

[8 marks]

Take moment about A

$Mg\sin\alpha \cdot l + \mu R\cos\alpha \cdot 2l = R\sin\alpha \cdot 2l$

~~resolve vertically: $Mg = \mu S + R$~~

~~horizontally: $\mu R =$~~

resolve horizontally $S = \mu R$

vertically $Mg = R + \mu^2 R$

$(R + \mu^2 R)\sin\alpha \cdot l + \mu R\cos\alpha \cdot 2l = R\sin\alpha \cdot 2l$

$(R + \mu^2 R)\sin\alpha + 2\mu R\cos\alpha = 2R\sin\alpha$

$$2\mu R\cos\alpha = 2R\sin\alpha - (R+\mu^2R)\sin\alpha$$

$$2\mu R\cos\alpha = \sin\alpha\,(2R - R - \mu^2R)$$

$$\frac{2\mu R}{R-\mu^2R} = \tan\alpha$$

$$\frac{2\mu}{1-\mu^2} = \tan\alpha$$

Answer $\tan\alpha = \dfrac{2\mu}{1-\mu^2}$

COMMENTARY

In this response the student provides a very clear method, stating where they are taking moments about and in which directions they are applying equilibrium of forces. Once they have the three relationships set up, they then proceed to eliminate variables to arrive at the correct final result. They are subsequently awarded all 8 marks for this question.

MARKS AWARDED: 8

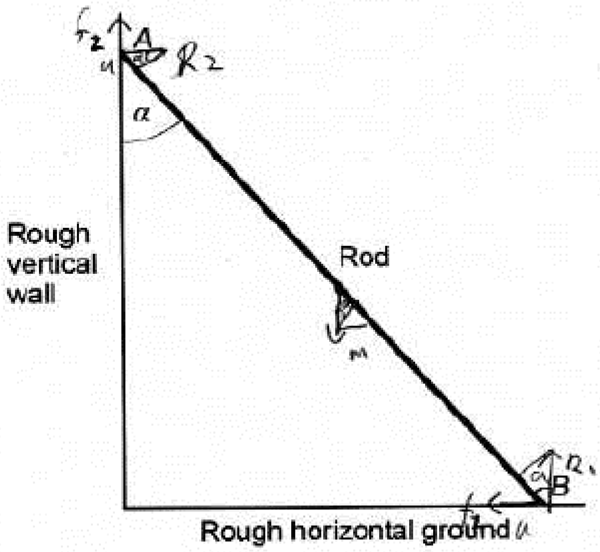
STUDENT B

RESPONSE

10 A uniform rod, AB, of mass M is in equilibrium, with one end in contact with a rough vertical wall. The other end is on rough horizontal ground.

The coefficient of friction between the wall and the rod is μ and the coefficient of friction between the ground and the rod is also μ.

The rod makes an angle α with the wall, as shown in the diagram.



Given that the rod is on the point of slipping, find tan α in terms of μ.

[8 marks]

$\begin{cases} f_2 + R_1 = mg \\ f_1 = R_2 \end{cases}$ $\begin{cases} f_2 = R_2 M \\ f_1 = R_1 M \end{cases}$

$\begin{cases} mg\frac{1}{2}l = \cos\alpha R_2 \cdot l \\ mg\frac{1}{2}l = \sin\alpha \cdot R_1 \cdot l \end{cases}$

$mg\frac{1}{2}l = \cos\alpha \cdot f_1 \cdot l$ $\qquad mg\frac{1}{2}\tan\alpha = mg\frac{1}{2}M$

$mg\frac{1}{2}l = \sin\alpha \frac{f_1}{M} \cdot l$ $\qquad \tan\alpha = M$

$mg\frac{1}{2} = \cos\alpha f_1$

$mg\frac{1}{2}M = f_1 \sin\alpha$

$\frac{mg\frac{1}{2}}{\cos\alpha} = \frac{mg\frac{1}{2}M}{\sin\alpha}$

$mg\frac{1}{2}\sin\alpha = mg\frac{1}{2}M\cos\alpha$

Answer $\tan\alpha = M$

COMMENTARY

In this response the student is awarded 2 marks (B1 and B1) for correct use of vertical and horizontal equilibrium of forces. They are not awarded the initial M1 mark is not awarded as an attempt at moments should include three terms when taken about A or B. As this is not awarded, the later m1 dependent method marks are then unavailable to be awarded.

MARKS AWARDED: 2

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