OXFORD
INTERNATIONAL
AQA EXAMINATIONS

INTERNATIONAL GCSE MATHEMATICS

Teaching guidance

(9260)

For teaching from September 2016 onwards
For International GCSE exams in June 2018 onwards

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Contents

General Information - Disclaimer

This teaching guidance will help you plan by providing examples of the content of the specification.

It is not, in any way, intended to restrict what can be assessed in the question papers based on the specification.

Questions will be set in a variety of formats including both familiar and unfamiliar contexts.

Examples given in this teaching guidance illustrate the type of questions which would be asked on a question paper. However, the wording and format used in this guidance do not always represent how questions would appear in a question paper. Questions in this guidance have not been through the same rigorous checking process used in our question papers.

All knowledge from the Key Stage 3 and Key Stage 4 programmes of study is subsumed into the content of the GCSE specification.

Subject content

Students can be said to have confidence and competence with mathematical content when they can apply it flexibly to solve problems.

The expectation is that:

- All students will develop confidence and competence with the content identified in the Core tier
- Only the more highly attaining students will be assessed on the content in the Extension tier.

The distinction between Core and Extension tier applies to the content statements only, not to the assessment objectives or to the mathematical formulae in the appendix.

1 Number – Structure and calculation

N1

Order positive and negative integers, decimals and fractions; use the symbols =, ≠, <, >, ⩽, ⩾

Teaching Guidance

Students should be able to:

- know and use the word integer and the equality and inequality symbols
- recognise integers as positive or negative whole numbers, including zero
- order positive and/or negative numbers given as integers, decimals and fractions, including improper fractions.

Notes

Including use of a number line.

Students should know the conventions of an open circle on a number line for a strict inequality and a closed circle for an included boundary.

See A23

Examples

1 Put these numbers in order starting with the smallest.

$1\frac{1}{2}$ 1.12 $-1\frac{1}{2}$ -1.12 $1\frac{2}{3}$ $1\frac{1}{8}$

2 Write 4.2, 4.02, 4.203 and 4.23 in ascending order.

3 Write these fractions in order of size, starting with the smallest.

$\frac{5}{6}$ $\frac{2}{3}$ $\frac{7}{9}$

4 Which of the improper fractions $\frac{18}{5}$, $\frac{17}{6}$ or $\frac{31}{10}$ is the greatest?

5 Which of these is closest to $\frac{1}{3}$?

0.35 $\frac{3}{10}$ 0.29 $\frac{1}{2}$

6 Write down the integer values of x where $3 < x \leqslant 8$

7 Put these numbers in ascending order.

$$\frac{3}{4} \qquad 0.83 \qquad \frac{4}{5} \qquad \frac{13}{20}$$

N2

Apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers - all both positive and negative; understand and use place value (eg when working with very large or very small numbers, and when calculating with decimals)

Teaching Guidance

Students should be able to:

- add, subtract, multiply and divide positive and negative numbers
- add, subtract, multiply and divide integers
- add, subtract, multiply and divide decimals
- add, subtract, multiply and divide fractions
- interpret a remainder from a division problem
- recall all positive number complements to 100
- recall all multiplication facts to 12×12 and use them to derive the corresponding division facts
- perform money and other calculations, writing answers using the correct notation
- multiply and divide a fraction by an integer, by a unit fraction and by a general fraction
- divide an integer by a fraction
- convert \$1 to 100 cents and vice versa in money calculations.

Notes

Students may use any algorithm for addition, subtraction, multiplication and division.

Questions will be set in a variety of contexts, both familiar and unfamiliar. For example, in household finance questions, students will be expected to know and understand the meaning of profit, loss, cost price, selling price, debit, credit, balance and interest rate.

See N7

Examples

1 Write down the place value of 8 in the answer to 2850×10

2 There are 75 students travelling in 16-seater mini-coaches.

If as many of the mini-coaches as possible are full, how many students travel in the mini-coach that is only partly full?

3 Four cards are numbered 3, 5, 7 and 8

Use each card once to make this calculation work.

……. ……. + ……. ……. = 158

4 The temperature falls 4°C from –3.5°C.

Work out the new temperature.

5 Here is a bank statement.

(a) Write down what you understand by the word ‘Balance’.

(b) Complete the statement.

Date	**Description**	**Credit**	**Debit**	**Balance**
	Starting balance			$63.50
12/12/2013	Cash	$120.00		…………..
16/12/2013	Gas bill		$102.50	…………..
17/12/2013	Electricity bill		$220.00	…………..

6 A builder employs seven bricklayers.

Each bricklayer earns $12.60 per hour.

They each work $37\frac{1}{2}$ hours per week.

The builder says he needs $33 075 each week to pay his bricklayers.

Use a calculator to check if he is correct.

7 A builder employs some bricklayers.

Each bricklayer works $37\frac{1}{2}$ hours per week.

He needs the bricklayers to work a total of at least 500 hours per week.

What is the minimum number of bricklayers he should employ?

8 Show that $-1\frac{3}{4} \times 4$ simplifies to an integer.

9 Show that $3\frac{5}{6} - 2\frac{1}{2} = 1\frac{1}{3}$

10 Show that $8 \div \frac{2}{3}$ simplifies to an integer.

N3 **Recognise and use relationships between operations,including inverse operations (eg cancellation to simplify calculations and expressions); use conventional notation for priority of operations, including brackets, powers, roots and reciprocals**

Teaching Guidance

Students should be able to:

- add, subtract, multiply and divide using commutative, associative and distributive laws
- understand and use inverse operations
- use brackets and the hierarchy of operations
- solve problems set in words.

Notes

Questions requiring these number skills could be set, for example, as a numerical part of a question testing time, fractions, decimals, percentages, ratio or proportion, interpreting graphs, using a formula in words or substituting into an algebraic expression, interpreting a statistical diagram or interrogating a data set.

Examples

1 Use all of the numbers 2, 5, 9 and 10, brackets and any operations to write a numerical expression equal to 3

2 A cup of coffee costs $1.30
A cup of tea costs $1.10
I buy three cups of coffee and two cups of tea.

How much change should I get from $10 ?

3 A cup of coffee costs $1.30
A cup of tea costs $1.10
I want to buy three cups of coffee and two cups of tea.
I have a voucher for one free cup of coffee with every two cups of coffee bought.

How much should I pay?

4 The data below shows the number of people in the seats in each row of a theatre.

19 17 14 15 18 17 12 8 4 3

The theatre has 200 seats.

How many empty seats are there?

5 The mean weight of 9 people is 79 kg
A 10th person is added, andthe mean weight increases by 1 kg

How heavy is the 10th person?

6 Luke says that $3 + 4 \times 5 = 35$

Is he correct?

Give a reason for your answer.

7 A coach firm charges \$300 to hire a coach, plus a rate per kilometre.

A group hires a coach and is charged a total of \$700 for a 200 kilometre journey.

What is the rate per kilometre?

9 125 people raise money for charity by running a marathon.
They raise \$5212.50 altogether.

Work out the mean amount raised per person.

10 The mean of this frequency distribution is 16

Data	Frequency
	10
15	43
20	21
25	11

Work out the missing data value.

11 Write $\frac{1}{\sqrt{121}}$ as a decimal.

Give your answer to 2 decimal places.

N4

Use the concepts and vocabulary of prime numbers, factors (divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation theorem

Teaching Guidance

Students should be able to:

- identify multiples, factors and prime numbers from lists of numbers
- write out lists of multiples and factors to identify common multiples or common factors of two or more integers
- write a number as the product of its prime factors and use formal (eg using Venn diagrams) and informal methods (eg trial and error) for identifying highest common factors (HCF) and lowest common multiples (LCM).

Notes

The unique factorisation theorem states that every integer greater than 1 is prime or can be written as the product of prime numbers.

Writing a number as the product of its prime factors including writing in index form.

Abbreviations will not be used in examination papers.

Examples

1 Which of the numbers 1, 6, 11, 12, 18, and 24 are factors of 24?

2 Write 60 as the product of its prime factors.

Give your answer in index form.

3 Envelopes are sold in packs of 18
Address labels are sold in packs of 30
Terry needs the same number of envelopes and address labels.

What is the smallest number of each pack he can buy?

4 a and b are prime numbers.

Work out a pair of values for *a* and *b* so that $4a - b$ is prime.

5 Write 1764 as a product of its prime factors.

N5

Use positive integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5

Teaching Guidance

Students should be able to:

- recall squares of numbers up to 15 × 15 and the cubes of 1, 2, 3, 4, 5 and 10, also knowing the corresponding roots
- calculate and recognise powers of 2, 3, 4 and 5
- calculate and recognise powers of 10
- understand the notation and be able to work out the value of squares, cubes and powers of 10
- recognise the notation $\sqrt{25}$
- solve equations such as $x^2 = 25$, giving both the positive and negative roots.

Notes

Students should know that $1000 = 10^3$ and 1 million $= 10^6$

Students should know that a square root can be negative when solving an equation.

Examples

1 Write down the values of $\sqrt{49}$, $\sqrt[3]{125}$, $\sqrt{2^3 + 2^3}$

2 Three numbers add up to 60
The first number is a square number.
The second number is a cube number.
The third number is less than 10

What could the numbers be?

3 Write down the value of

(a) 8^2

(b) 4^3

(c) 10^5

4 Write 64 as

(a) a square of an integer.

(b) the cube of an integer.

N5e **Use positive integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5**

Teaching Guidance

Students should be able to:

- identify between which two integers the square root of a positive number lies
- identify between which two integers the cube root of a positive number lies.

Examples

1 Between which two integers does the square root of 150 lie?

2 Between which two integers does the cube root of 100 lie?

N6 | **Index laws for multiplication and division using integer powers**

Teaching Guidance

Students should be able to:

- use index laws for multiplication and division of integer powers
- calculate with positive and negative integer powers.

Notes

Students will be expected to apply index laws to simplify algebraic expressions.

See A6

Examples

1 Write

(a) $7^5 \times 7^3$ as a single power of 7

(b) $9^{12} \div 9^5$ as a single power of 9

(c) $(2^5)^3$ as a single power of 2

2 Work out the value of $2^6 \div 2^{10}$, giving your answer as a fraction.

3 Work out the value of 2^{-3}

4 Amy writes that $6^{10} \div 6^2 = 6^5$

Explain what Amy has done wrong.

5 Work out the value of $\dfrac{2^3 \times 2^6}{2^5}$

(a) as a power of 2

(b) as an integer

6 Simplify $a^6 \times a^2$

N6e

Index laws for multiplication and division using fractional powers

Teaching Guidance

Students should be able to:

- calculate values using fractional indices
- use index laws for multiplication and division of fractional indices.

Notes

Students will be expected to apply index laws to simplify algebraic expressions.

See A6

Examples

1 Work out the value of $8^{\frac{1}{3}}$

2 Show that $9^{-\frac{1}{2}}$ simplifies to $\frac{1}{3}$

3 Simplify $\sqrt{x^{\frac{7}{2}} \times x^{-\frac{3}{2}}}$

4 Work out the value of $16^{-\frac{3}{4}}$

You **must** show your working.

5 Work out x if $27^{x} = 81^{-\frac{1}{4}}$

N7	**Calculate exactly with fractions**

Teaching Guidance

Students should be able to:

- identify equivalent fractions
- write a fraction in its simplest form
- simplify a fraction by cancelling all common factors, using a calculator where appropriate, for example, simplifying fractions that represent probabilities
- convert between mixed numbers and improper fractions
- compare fractions
- compare fractions in statistics and geometry questions.
- add and subtract fractions by writing them with a common denominator
- convert mixed numbers to improper fractions and add and subtract mixed numbers
- give answers in terms of π and use values given in terms of π in calculations.

Notes

See N2, G16, G18

Examples

1 Which of these fractions $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{12}$ are greater than $\frac{2}{3}$?

You **must** show your working.

2 Which of these fractions $\frac{2}{5}$, $\frac{9}{20}$ or $\frac{7}{10}$ is closest to $\frac{1}{2}$?

You **must** show your working.

3 Peter scores 64 out of 80 in a test.

Write this score as a fraction in its simplest form.

4 Write down a fraction between $\frac{5}{7}$ and $\frac{6}{7}$

5 Write down an improper fraction with a value between 3 and 4

6 From inspection of a bar chart:

What fraction of the boys preferred pizza?

Give your answer in its simplest form.

7 From inspection of a pie chart:

What fraction of the vehicles were cars?

Give your answer in its simplest form.

8 Trading Standards staff inspect 80 bags of apples to check that they weigh 1 kg as stated.

The diagram shows the weight of the bags **under** 1 kg (diagram given)

What fraction of the bags were under 1 kg?
Give your answer in its simplest form.

9 Show that

(a) $\frac{2}{5} + \frac{3}{4} = 1\frac{3}{20}$

(b) $3\frac{2}{3} - 1\frac{1}{2} = 2\frac{1}{6}$

10 In an experiment to test reaction times, Alex took $\frac{1}{8}$ of a second to react and Ben took $\frac{3}{20}$ of a second to react.

Who reacted more quickly, and by how much?

11 Sally is cycling home, a distance of $6\frac{1}{3}$ kilometres.

After $4\frac{3}{4}$ kilometres she has a puncture and has to push her bike the rest of the way.

How far does she push her bike?

N7e | **Calculate exactly with surds; manipulation and simplification of surds including rationalising a denominator**

Teaching Guidance

Students should be able to:

- simplify surds
- rationalise a denominator
- simplify expressions using the rules of surds
- expand brackets where the terms may be written in surd form
- solve equations which may be written in surd form.

Examples

1 Show that $\sqrt{28} + \sqrt{63}$ can be written in the form $p\sqrt{7}$ where p is an integer.

2 Simplify

(a) $(\sqrt{11})^2$

(b) $\sqrt{192} \div \sqrt{12}$

3 (a) Show that $\sqrt{27} \times \sqrt{3}$ is an integer.

(b) Rationalise the denominator to show that $\dfrac{35}{\sqrt{7}}$ can be written as $a\sqrt{7}$ where a is an integer.

4 Show that $\sqrt{20} = 2\sqrt{5}$

5 Show that $(\sqrt{2} + \sqrt{10})^2$ can be written in the form $a + b\sqrt{5}$ where a and b are integers.

6 Show that $(\sqrt{27} + 3)(\sqrt{6} - \sqrt{2})$ can be simplified to $a\sqrt{2}$ where a is an integer.

7 Work out the value of x given that $\dfrac{\sqrt{x} \times \sqrt{50}}{5} = 4\sqrt{5}$

8 Show that $\dfrac{5}{3\sqrt{2}} - \dfrac{3\sqrt{2}}{4}$ simplifies to $\dfrac{\sqrt{2}}{12}$

9 Show that $\dfrac{26}{4 - \sqrt{3}}$ simplifies to $a + b\sqrt{c}$ where a, b and c are integers

N8

Calculate with and interpret standard form $A \times 10^n$, where $1 \leqslant A < 10$ and n is an integer

Teaching Guidance

Students should be able to:

- know, use and understand the term standard form
- write an ordinary number in standard form
- write a number written in standard form as an ordinary number
- order and calculate with numbers written in standard form
- solve simple equations where the numbers are written in standard form
- interpret calculator displays
- use a calculator effectively for standard form calculations
- solve standard form problems with and without a calculator.

Examples

1 Write in standard form

(a) 379.4

(b) 0.0712

2 Write as ordinary numbers

(a) 2.65×10^5

(b) 7.08×10^{-3}

3 Write one quarter of a million in standard form.

4 Write these numbers in ascending order

14 485 $\quad$ 1.45×10^4 $\quad$ 1.45×10^3

5 Work out the value of $(2.8 \times 10^9) \div (4 \times 10^5)$

Give your answer in standard form.

6 Solve $(2.4 \times 10^7)x = 1.44 \times 10^9$

Give your answer in standard form.

N9

Use language and notation of sets including n(A), A', $A \cup B$, $A \cap B$, ξ ; understand and use Venn diagrams to solve problems

Teaching Guidance

Students should know

- understand that n(A) means the number of elements in A
- understand that A' means the set with elements **not** in A
- understand that $A \cup B$ means the set with elements in A or B or both
- understand that $A \cap B$ means the set with elements in A and B
- understand and use a universal set in different situations
- understand a Venn diagram consisting of a universal set containing other sets, which may or may not intersect
- shade areas on a Venn diagram involving sets, which may or may not intersect
- solve problems given a Venn diagram
- solve problems, where a Venn diagram approach is a suitable strategy to use but a diagram is not given in the question.

Examples

1 ξ is the set of integers from 1 to 30 inclusive.

A is the set of prime numbers between 1 and 30

B is the set of odd numbers between 2 and 10

(a) Work out n(A)

(b) Work out B'

(c) Work out $A \cup B$

(d) Work out $A \cap B$

N10 | **Use calculators effectively and efficiently including trigonometrical functions**

Teaching Guidance

Students should be able to:

- calculate using a scientific calculator

Examples

1 Calculate

(a) 284×56.8

(b) $\sqrt{4692.25} - 2.4^4$

(c) $\tan^{-1}\frac{16}{34}$

N11

Round numbers and measures to an appropriate degree of accuracy (eg to a specified number of decimal places or significant figures); apply and interpret limits of accuracy; use estimation to work out approximate answers to calculations.

Teaching Guidance

Students should be able to:

- round numbers to the nearest whole number, 10, 100 or 1000
- round numbers to a specified number of decimal places
- round numbers to a specified number of significant figures
- know that measurements using real numbers depend on the choice of unit
- recognise that measurements given to the nearest whole unit may be inaccurate by up to one half in either direction.
- make sensible estimates of a range of measures in real-life situations, for example estimate the height of a man
- evaluate results obtained
- use approximation to estimate the value of a calculation
- work out the value of a calculation and check the answer using approximations.

Notes

Including appropriate rounding for questions set in context.

Students should know not to round values during intermediate steps of a calculation.

Students should know that some answers need to be rounded up and some need to be rounded down.

Students should know that some answers are inappropriate without some form of rounding, for example 4.2 buses.

Students should know that when using approximations for estimating answers, numbers should be rounded to one significant figure before the estimating is done.

Examples

1 How many 40-seater coaches are needed to carry 130 students?

2 120 people take their driving test in a week.

71 pass.

Work out the percentage of people who pass.

Give your answer to one decimal place.

3 1127 people pass their driving test in 39 weeks.

Calculate the mean number of students who pass in one week.

Give your answer to one significant figure.

4 Estimate the height of a building and use this to estimate the number of pieces of 2m drainpipe needed (diagram given).

5 (a) Work out the exact value of 9.8×109

(b) Use approximations to decide whether your answer to part (a) is sensible.

N11e **Calculate and use upper and lower bounds**

Teaching Guidance

Students should be able to:

- write down the maximum or minimum figure for a value rounded to a given accuracy
- combine upper or lower bounds appropriately to achieve an overall maximum or minimum for a situation
- work with practical problems involving bounds including in statistics. For example, finding the midpoint of a class interval, such as $10 < t \leqslant 20$, in order to estimate a mean.

Notes

For example, the maximum value of $a - b$ is obtained from use of the maximum value for a and the minimum value for b.

Upper bounds do not necessarily require use of recurring decimals. For example, if the answer to the nearest integer is 7, the maximum could be given as 7.5 or $7.4\dot{9}$

If this value of 7 represented \$7, \$7.49 would be expected for the maximum.

For continuous variables, students may be asked for the lower and upper limits rather than the minimum and maximum values.

Examples

1 The current men's 100 metre sprint world record is 9.58 seconds.

(a) Why is this unlikely to be an exact time?

(b) What is the shortest time this could have been?

2 The mean height of Nick's children is exactly 1.15 m
Each child’s height is measured to the nearest cm

What is the greatest possible sum of the four children's heights?

3 In 2015 Nita bought a car for \$10 000 to the nearest \$100
In 2016 the car went down in value by 15% to the nearest 1%
In 2017 the car went down in value by a further 13% to the nearest 1%

What was the highest possible value of the car by the end of 2017?

4 Ben is planning a car journey.

He estimates the distance to be 90 kilometres to the nearest 5 kilometres.

He hopes to travel at an average speed of 60 km per h to the nearest 10 km per h.

Work out his minimum expected journey time in hours and minutes.

5 A rectangle has a length of 3.4 cm and a width of 5.7 cm

The length and width are given to 1 decimal place.

Work out the minimum possible area.

N12 **Understand and use equivalent fractions, understand and use percentages, convert between fractions, terminating decimals and percentages**

Teaching Guidance

Students should be able to:

- convert between fractions and decimals using place value
- compare the value of fractions and decimals
- convert values between percentages, fractions and decimals in order to compare them, for example with probabilities.

Notes

Including ordering.

Examples

1 Show that $\frac{7}{8} = 0.875$.

2 Write 0.28 as a fraction in its lowest terms.

3 Write $\frac{11}{4}$ as a decimal.

4 Write 35% as

(a) a decimal

(b) a fraction in its simplest form.

N12e

Convert between fractions and recurring decimals

Teaching Guidance

Students should be able to:

- convert recurring decimals into fractions
- convert fractions into recurring decimals
- use formal algebraic proofs to convert recurring decimals into fractions.

Examples

1 Write $0.\dot{7}\dot{2}$ as a fraction in its lowest terms.

You **must** show your working.

2 Write $\frac{7}{90}$ as a recurring decimal.

3 Put these numbers in ascending order $\frac{4}{11}$, 0.4, $\frac{3}{10}$, $0.\dot{3}\dot{9}$

4 Show that $0.4\dot{3}\dot{6} = \frac{24}{55}$

5 Put these probabilities in order, starting with the least likely.

A 65%

B 0.7

C $\frac{2}{3}$

N13 **Interpret fractions and percentages as operators**

Teaching Guidance

Students should be able to:

- calculate a fraction of a quantity
- calculate a percentage of a quantity
- use fractions, decimals or percentages to find quantities
- use fractions, decimals or percentages to calculate proportions of shapes that are shaded
- use fractions, decimals or percentages to calculate lengths, areas or volumes
- understand and use unit fractions as multiplicative inverses
- multiply and divide a fraction by an integer, by a unit fraction and by a general fraction
- interpret a fraction, decimal or percentage as a multiplier when solving problems
- use fractions, decimals or percentages to interpret or compare statistical diagrams or data sets
- convert between fractions, decimals and percentages to find the most appropriate method of calculation in a question; for example, 62% of \$80 is $0.62 \times$ \$80 and 25% of \$80 is \$80 $\div$ 4

Notes

Students should understand that, for example, multiplication by $\frac{1}{5}$ is the same as division by 5

Examples

1 A rectangle measures 3.2 cm by 6.8 cm
It is cut into four congruent smaller rectangles.

Work out the area of a small rectangle.

2 Cubes of edge length 1 cm are put into a box.
The box is a cuboid of length 5 cm, width 4 cm and height 2 cm

How many cubes are in the box if it is half full?

3 In a school there are 600 students and 50 teachers.

15% of the students are left-handed.

12% of the teachers are left-handed.

How many left-handed students and teachers are there altogether?

4 Work out $\frac{3}{8}$ of 56

5 Circle the calculations that would find 45% of 400

A 0.45×400

B $\frac{1}{45} \times 400$

C $\frac{45}{100} \times 400$

D $\frac{400}{4} \times 5$

E 45×4

6 Work out 62% of \$70

7 In school A, 56% of the 750 students are girls.

In school B, $\frac{4}{9}$ of the 972 students are girls.

Which school has the greater number of girls and by how many?

8 The cash price of a leather sofa is \$700

Credit terms are a 20% deposit plus 24 monthly payments of \$25

Calculate the difference between the cash price and the credit price.

N14

Express one quantity as a fraction or percentage of another, where the fraction is less than 1 or greater than 1

Teaching Guidance

Students should be able to:

- work out one quantity as a fraction or decimal or percentage of another quantity
- use a fraction or percentage of a quantity to compare proportions.

Notes

Questions may use a mixture of units.

Answers will be expected to be in their simplest form.

Examples

1 In a class of 32 students, 15 are boys.

What fraction of the class are girls?

2 The average mark scored in an exam was 54

(a) Jack scored a mark of 42

Write Jack's mark as a fraction of the average mark.

(b) Bel's mark as a fraction of the average mark was $\frac{3}{2}$

What does this tell you about Bel's mark?

3 A television programme lasted 90 minutes, including 12 minutes of advertising.

Another programme lasted 60 minutes, including 10 minutes of advertising.

Which programme had a greater proportion of advertising?

You must show your working.

N15 **Solve problems involving percentage change, including increase/decrease, simple interest and compound interest**

Teaching Guidance

Students should be able to:

- calculate a percentage increase or decrease
- solve percentage increase and decrease problems. For example, use 1.12 × Q to calculate a 12% increase in the value of Q and 0.88 × Q to calculate a 12% decrease in the value of Q
- solve simple interest problems
- solve compound interest problems.

Examples

1 Chris earns $285 per week.

He gets a 6% pay rise.

How much per week does he earn now?

2 Paving slabs cost $3.20 each.
A supplier offers '20% off on an order of more than $300'.

How much will it cost to buy 100 paving slabs?

3 Before a storm, a pond held 36 000 litres of water.
After the storm, the volume of the pond increased by 12%

How many litres of water does the pond hold **after** the storm?

4 The mean price of four train tickets is $25
All prices are increased by 10%

What is the total cost of the four tickets **after** the price increase?

5 A car increases in speed from 50 mph to 70 mph.

Work out the percentage increase.

6 £5000 is invested at 9.65% compound interest.

Work out the value of the investment after six years.

N15e

Reverse percentage problems; knowledge and use of the compound interest formula

value of investment = $P(1 + \frac{r}{100})^n$ where P is the amount invested, r is the percentage rate of interest and n is the number of years of compound interest

Teaching Guidance

Students should be able to:

- calculate reverse percentages
- solve compound interest problems involving the compound interest formula.

Examples

1 Attendance at a football match is 48400

This is a 10% increase on the attendance at the last game.

What was the attendance at the last game?

2 The value of my car has decreased by 15% of the price I paid one year ago.

It is now valued at $17 340

How much did I pay for the car one year ago?

3 $2500 is invested.

After 6 years, the value of the investment is $2985.13

Work out the interest rate.

Give the answer to 1 decimal place.

N16 **Use ratio notation, including reduction to simplest form and links to fraction notation**

Teaching Guidance

Students should be able to:

- understand the meaning of ratio notation
- interpret a ratio as a fraction
- simplify ratios to their simplest form $a : b$ where a and b are integers
- write a ratio in the form $1 : n$ or $n : 1$

Examples

1 There are 6 girls and 27 boys in an after-school computer club.

Write the ratio of girls : boys in its simplest form.

2 Write the ratio 15 : 8 in the form $n : 1$

3 The ratio of left-handed people to right-handed people in a class is 2 :19

What fraction of the people are right-handed?

N17 **Divide a quantity in a given ratio**

Teaching Guidance

Students should be able to:

- use ratio to solve, for example geometrical, statistical, and number problems
- use ratio to solve word problems.

Examples

1 Ann, Bob and Carl share \$480 in the ratio 1 : 4 : 3

How much should each of them receive?

2 The ratio of boys to girls in a school is 5 : 6

There are 468 girls in the school.

How many students are there altogether?

3 Jen and Kim pay for a present for their mum in the ratio 7 : 9

Jen pays \$21

How much did the present cost?

N18 **Apply ratio to solve problems**

Teaching Guidance

Students should be able to:

- use ratios in the context of geometrical problems, for example similar shapes, scale drawings and problem solving involving scales and measures
- interpret a ratio in a way that enables the correct proportion of an amount to be calculated.
- use ratio to solve, for example geometrical, statistical, and number problems
- use ratio to solve word problems using informal strategies or using the unitary method of solution.

Examples

1 A recipe for fruit cake uses sultanas and raisins in the ratio 5 : 3

How many grams of sultanas should be used with 150 grams of raisins?

2 The number of coins in two piles are in the ratio 5 : 3

The coins in the first pile are all 2 p coins.

The coins in the second pile are all 5 cent coins.

There is 45 cents in the second pile.

How much money is in the first pile?

3 From a bar chart showing the results for girls (small amount of discrete data):

The ratio of the means for the girls and the boys is 1 : 2

Draw a possible bar chart for the boys.

N19 | **Use common measures of rate, including calculating rates of pay and best-buy problems**

Teaching Guidance

Students should be able to:

- calculate rates in different contexts
- solve best-buy problems using informal strategies or using the unitary method of solution.

Examples

1 Ann works for 40 hours and is paid \$464.

Work out her rate of pay per hour.

2 A 500 litre tank is filled at a rate of 80 litres per minute.

How long does it take to fill the tank?
Give your answer in minutes and seconds.

3 Cola is sold in two packs.
A pack of 20 cans costs \$4.99 and a pack of 12 cans costs \$2.75

Which pack is better value for money?
You **must** show your working.

N20

Solve problems involving direct and inverse proportion including repeated proportional change

Teaching Guidance

Students should be able to:

- use proportion to solve problems using informal strategies or the unitary method of solution
- use direct proportion to solve geometrical problems
- calculate an unknown quantity from quantities that vary in direct proportion or inverse proportion
- set up and use equations to solve problems involving direct proportion or inverse proportion
- relate algebraic solutions to graphical representation of the equations
- sketch an appropriately shaped graph (which may be partly or entirely non-linear) to represent a real-life situation
- choose the graph that is sketched correctly from a selection of alternatives
- recognise the graphs that represent direct and inverse proportion.

Notes

Graphs may be given as sketches or as accurate drawings.

Examples

1 You can buy x pencils for 80p

How much will it cost to buy y pencils?

2 Two men can mow a meadow in two hours.

How long would it take three men to mow the meadow, assuming they work at the same rate?

3 Tom visits friends in his car.
If he averages 60 miles per hour, the journey takes $2\frac{1}{2}$ hours.

How long will his journey take if he averages 40 miles per hour?

4 An aircraft leaves Berlin when Helga's watch reads 07.00 and lands in New York when her watch reads 14.00

Helga does not change the time on her watch.

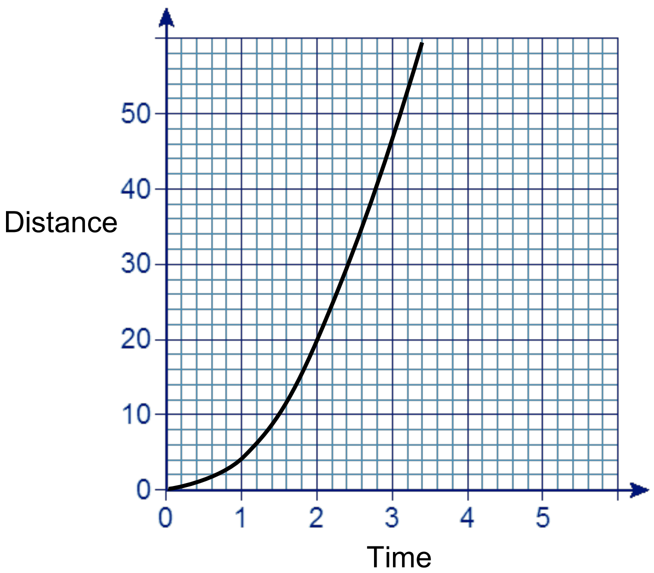
The distance from Berlin to New York is 5747 kilometres.

Assuming that the aircraft flies at a constant speed, how far does the aircraft fly between the hours of 09.00 and 11.00?

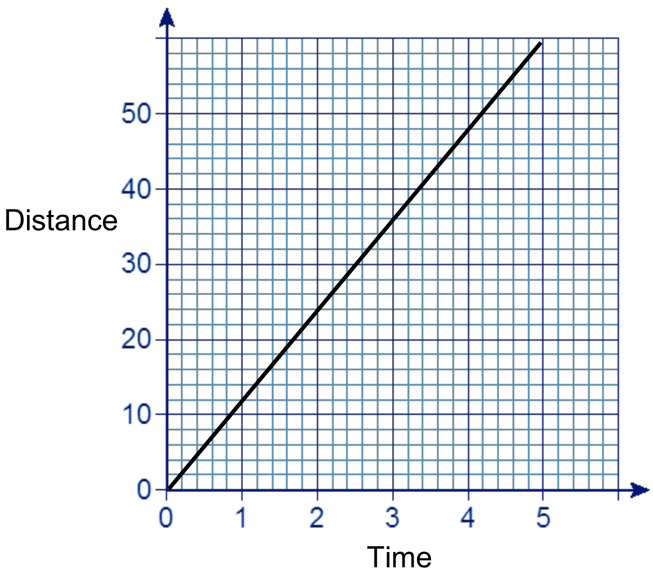
5 Match this statement to a graph.

The distance travelled at a steady speed is directly proportional to the time taken.

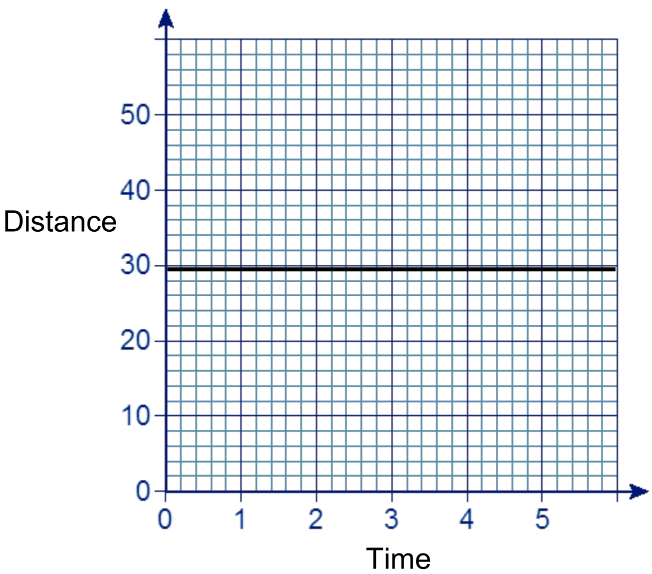
Graph A



Graph B



Graph C



N20e **Exponential growth and decay**

Teaching Guidance

Students should be able to:

- model growth and decay problems mathematically
- solve growth and decay problems.

Notes

Understand that, for example, the number of fish in a small pond cannot continue to grow indefinitely and that certain assumptions will be made or used when modelling, for example a 10% increase in population per year.

Examples

1 A population is decaying at the rate of 5% per year.

Write down a calculation to work out the population after 6 years.

4 Algebra – Notation and manipulation

A1 **Use letters to express generalised numbers and express basic arithmetic processes algebraically**

Teaching Guidance

Students should be able to:

- use notation and symbols correctly
- understand that letter symbols represent definite unknown numbers in equations, defined quantities or variables in formulae, and in functions they define new expressions or quantities by referring to known quantities.

Notes

It is expected that answers will be given in their simplest form without an explicit instruction to do so.

Students will be expected to know the standard conventions. For example, $2x$ for $2 \times x$ and $\frac{1}{2}x$ or $\frac{x}{2}$ for $x \div 2$

$x2$ is not acceptable for $2 \times x$

Examples

1 $\$x$ is shared equally between seven people.

How much does each person receive?

2 Write an expression for the total cost of 3 kg of apples at a dollars per kg and 5 kg of pears at b dollars per kg.

A2

Substitute numbers for words and letters in formulae and transform simple formulae

Teaching Guidance

Students should be able to:

- understand and use formulae from mathematics and other subjects expressed initially in words and then using letters and symbols. For example, formula for area of a triangle, area of a parallelogram, area of a circle, volume of a prism, conversions between measures, wage earned = hours worked × hourly rate + bonus
- substitute numbers into a formula
- change the subject of a simple formula.

Notes

Questions will include geometrical formulae and questions involving measures.

Questions will include formulae for generating sequences and formulae in words using a real-life context (for example formula for cooking a turkey) and formulae out of context (for example substitute positive and negative numbers into expressions such as $3x^2 + 4$, $2x^3$ and $\frac{5(x - 3y)}{2}$)

Unfamiliar formulae will be given in the question.

Examples

1 To change a distance given in miles, m, to a distance in kilometres, k, we use this rule.

First multiply by 8 then divide by 5

Write this rule as a formula and use it to change 300 miles into kilometres.

2 Write down the first three terms of a sequence where the nth term is given by $n^2 + 4$

3 Use the formula 'wage earned = hours worked × hourly rate + bonus' to calculate the wage earned when Sarah works for 30 hours at \$8.50 an hour and receives a bonus of \$46

4 Convert 25°C to °F using the formula $F = \frac{9}{5}C + 32$

5 When $a = 5$, $b = -7$ and $c = 8$, work out the value of $\frac{a(b + 3)}{c}$

6 Work out the perimeter of a semicircle of diameter 8 cm

7 Rearrange $y = 2x + 3$ to make x the subject.

8 Rearrange $C = 2\pi r$ to make r the subject.

A2e **Transform complex formulae including when the subject appears twice**

Teaching Guidance

Students should be able to:

- change the subject of a complex formula.

Examples

1 Rearrange $y = \dfrac{3x+2}{1-5x}$ to make x the subject.

2 Rearrange $x + 2w = 5(w - 2)$ to make w the subject.

3 Rearrange $y = \sqrt{\dfrac{5x+1}{y}}$ to make w the subject.

A3 **Understand and use the concepts and vocabulary of expressions, equations, formulae, identities, inequalities, terms and factors**

Teaching Guidance

Students should be able to:

- understand phrases such as ‘form an equation’, ‘use a formula’, ‘write down a term’, ‘write an expression’ and ‘prove an identity’ when answering a question
- recognise that, for example, $5x + 1 = 16$ is an equation
- recognise that, for example, $V = IR$ is a formula
- recognise that $x + 3$ is an expression
- recognise that $(x + 2)^2 \equiv x^2 + 4x + 4$ is an identity
- recognise that $2x + 5 < 16$ is an inequality
- write an expression
- know the meaning of the word ‘factor’ for both numerical work and algebraic work.

Notes

This will be implicitly and explicitly assessed.

Examples

1 Write an expression for the number that is one sixth as big n.

2 Write an expression for the number that is three less than half of x.

3 Given an equation, a formula, an inequality and an expression, match each of them to the correct word.

4 Write down six different factors of the expression $5ab$

6 Chloe is x years old.

Her sister is three years older.

Her brother is twice as old as Chloe.

The sum of their ages is 67 years.

(a) Write an expression, in terms of x, for her sister's age.

(b) Form an equation in x to work out Chloe's age.

7 Two angles have a difference of 30°

Together they form a straight line.

The smaller angle is x°

(a) Write down an expression, in terms of x, for the larger angle,.

(b) Work out the value of x.

A4	**Collecting like terms and expanding brackets up to expanding products of two linear expressions**

Teaching Guidance

Students should be able to:

- understand that algebra can be used to generalise the laws of arithmetic
- manipulate an expression by collecting like terms
- write expressions to solve problems
- multiply a single term over a bracket, for example, $a(b + c) = ab + ac$
- multiply two linear expressions, such as $(x \pm a)(x \pm b)$ and $(cx \pm a)(dx \pm b)$, for example $(2x + 3)(3x - 4)$
- know the meaning of and be able to simplify, for example $3x - 2 + 4(x + 5)$

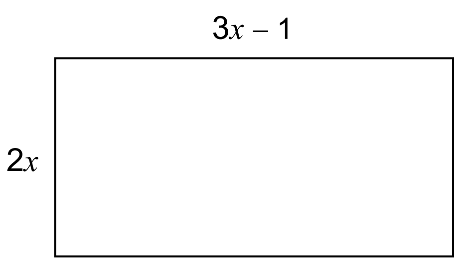
Examples

1 Multiply out and simplify $3(a - 4) + 2(2a + 5)$

2 Multiply out $6(w - 2y)$

3 Multiply out and simplify $(3a - 2b)(2a + b)$

4 This rectangle has dimensions as shown.



Not drawn accurately

The perimeter of the rectangle is 68 centimetres.

Use this information to form and solve an equation to work out the dimensions of the rectangle.

5 The expression $7(x + 4) - 3(x - 2)$ simplifies to $a(2x + b)$

Work out the values of a and b.

A4e | **Expanding products of two or three binomials**

Teaching Guidance

Students should be able to:

- multiply two or more binomial expressions.

Examples

1 Multiply out and simplify $(2x + 3)(3x - 4)(5x + 6)$

2 Multiply out $(x^2 + 6)(4x - 5y)$

3 Multiply out and simplify $(x - 5)^3$

A5

Taking out common factors, factorising quadratic expressions of the form $x^2 + bx + c$; including the difference of two squares

Teaching Guidance

Students should be able to:

- factorise algebraic expressions by taking out common factors
- know the meaning of and be able to factorise, for example $3x^2y - 9y$ or $4x^2 + 6xy$
- factorise quadratic expressions using the sum and product method, or by inspection
- factorise quadratics of the form $x^2 + bx + c$
- factorise expressions written as the difference of two squares of the form $x^2 - a^2$

Examples

1 Factorise $6w - 8y$

2 Factorise $12x^2 + 8x$

3 Factorise $15xy^2 - 2x^2y$

4 Factorise $x^2 - 7x + 10$

5 Factorise

(a) $y^2 - 9$

(b) $k^2 - m^2$

A5e

Factorising quadratic expressions of the form
$ax^2 + bx + c$ including the difference of two squares

Teaching Guidance

Students should be able to:

- factorise quadratic expressions of the form $ax^2 + bx + c$
- factorise expressions written as the difference of two squares of the form $a^2x^2 - b^2$

Examples

1 (a) Factorise $2n^2 + 5n + 3$

(b) Hence, or otherwise, write 253 as the product of two prime factors.

2 Factorise $3n^2 + 7n - 6$

3 Factorise $4x^2 - 9$

A6	**Index laws for multiplication and division using integer powers**

Teaching Guidance

Students should be able to:

- use the index laws for multiplication and division of integer powers.
- simplify algebraic expressions, for example by cancelling common factors in fractions or using index laws.

Examples

1 (a) Simplify $x^2 \times x^4$

(b) Simplify $x^{16} \div x^4$

2 Simplify $\dfrac{18x^5y}{3x}$

3 Simplify fully $\dfrac{x^5 \times x^7}{x^4}$

A6e **Index laws for multiplication and division using fractional powers**

Teaching Guidance

Students should be able to:

- use the index laws for multiplication and division of fractional powers.

Examples

1 (a) Simplify $x^{\frac{1}{2}} \times x^{-\frac{3}{2}}$

(b) Simplify $x^{\frac{1}{3}} \div x^{\frac{1}{2}}$

A7

Manipulation of rational expressions: use of + – × ÷ for algebraic fractions with denominators being numeric

Teaching Guidance

Students should be able to:

- manipulate rational expressions with denominators being numeric
- simplify rational expressions by cancelling.

Examples

1 Write as a single fraction $\frac{x}{3} + \frac{x}{2}$

2 Write as a single fraction $\frac{x}{5} \times \frac{x}{2}$

3 Write as a single fraction $\frac{a+2}{3} - \frac{a}{4}$

4 Simplify $\frac{4x + 12}{2}$

A7e

Manipulation of rational expressions: use of + – × ÷ for algebraic fractions with denominators being linear or quadratic

Teaching Guidance

Students should be able to:

- manipulate rational expressions with denominators being linear or quadratic
- simplify rational expressions by factorising.

Examples

1 Write as a single fraction $\dfrac{5}{2x} - \dfrac{1}{x}$

2 Write as an integer $\dfrac{6}{x+2} \div \dfrac{3}{2x+4}$

3 Write as a single fraction $\dfrac{a}{a+3} + \dfrac{1}{a-2}$

4 Write as a single fraction $\dfrac{2x^2+5x-3}{x^2-9}$

5 Simplify fully $\dfrac{6x^2+7x-3}{16x^2-1} \div \dfrac{2x^3+3x^2}{4x+1}$

6 Simplify fully $\dfrac{5x}{(x+1)(x-4)} - \dfrac{4}{(x-4)}$

7 Simplify fully $\dfrac{x^4-5x^3+6x^2}{x^4-13x^2+36}$

A8 **Argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments**

Teaching Guidance

Students should be able to:

- show that two expressions are equivalent
- use identities, including equating coefficients
- use algebraic expressions to support an argument or verify a statement.

Notes

Arguments may use knowledge of odd and even, for example odd $\times$ even = even

Students should understand that, for example, if n is an integer then $2n$ is even and $2n + 1$ is odd.

Students should be familiar with the term 'consecutive'.

Examples

1 Work out the values of a and b in the identity $2(ax - 5) + 3(5x + b) \equiv 21x + 2$

2 Show that $3(a - 4) + 2(2a + 5) + 9$ and $7(a + 1)$ are equivalent.

3 w is an even number.

Explain why $(w - 1)(w + 1)$ will always be odd.

4 Sam says that when $m > 1$, $m^3 + 2$ is never a multiple of 3

Give an example to show that she is wrong.

A8e **Argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments to include proofs**

Teaching Guidance

Students should be able to:

- construct rigorous proofs to validate a given result.

Notes

Arguments may use knowledge of odd and even, for example odd $\times$ even = even

Students should understand that, for example, if n is an integer then $2n$ is even and $2n + 1$ is odd.

Students should be familiar with the term ‘consecutive’.

Examples

1 Alice says that the sum of three consecutive numbers will always be even.

Explain why she is wrong.

2 Explain why the product of three consecutive positive integers must always be a multiple of 6

3 n is a positive integer.

Explain why $n(n + 1)$ must be an even number.

4 Prove that the square of any odd number is always one more than a multiple of 8

5 (a) Show that $\dfrac{2x+5}{3} + \dfrac{3x-1}{2}$ simplifies to $\dfrac{13x+7}{6}$

(b) Hence solve $\dfrac{2x+5}{3} + \dfrac{3x-1}{2} = 8$

6 Prove that $(2x + 3)^2 - 3x(x + 4)$ is always positive.

A9 **Interpret simple expressions as functions with inputs and outputs**

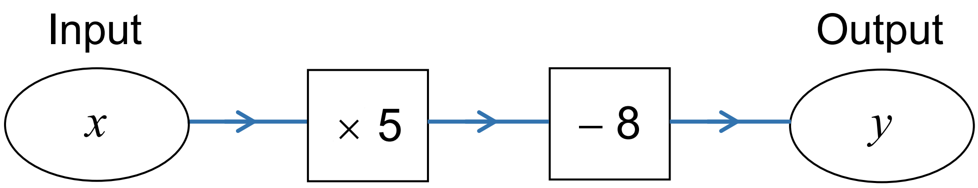
Teaching Guidance

Students should be able to:

- understand and use number machines
- interpret an expression diagrammatically using a number machine
- interpret the operations in a number machine as an expression or function.

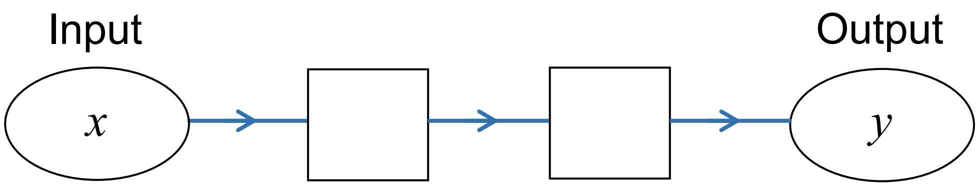
Examples

1

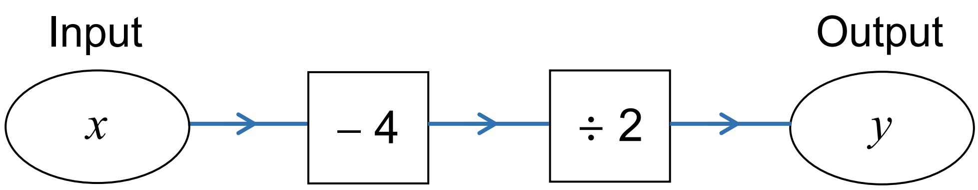


Write down the output y as an expression in terms of x.

2 Complete the number machine so that $y = 3x + 7$



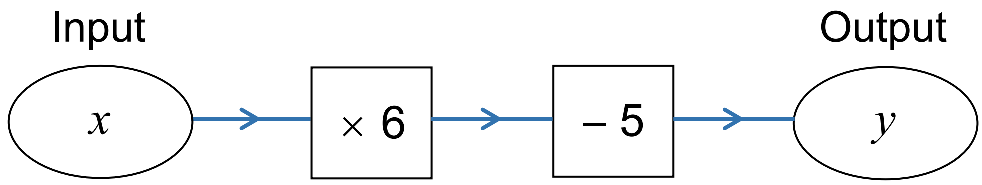
3



(a) Work out the output when the input is 1

(b) Work out the input when the output is 5

4 Work out the input x as an expression in terms of y.



A9e

Definition of a function, use function notation of the form f(x) = … , understand and use the terms domain and range, understand and find the composite function fg and the inverse function f^{-1}

Teaching Guidance

Students should be able to:

- understand that a function is a relationship between two sets of values
- understand and use function notation, for example f(x)
- substitute values into a function, knowing that, for example f(2) is the value of the function when $x = 2$
- solve equations that use function notation
- define the domain of a function
- work out the range of a function
- understand, interpret and use the composite function fg(x)
- understand, interpret and use the inverse function $f^{-1}(x)$

Examples

1 Given that $f(x) = 4x - 5$ work out

(a) f(–6)

(b) f(0.5)

2 $f(x) = 3x + 2$

Solve $f(x) = 0$

3 $f(x) = x^2 + 3x - 6$

(a) Write down an expression for f(2x)

(b) Solve $f(2x) = 0$

Give your answer to 2 decimal places.

4 Given that $f(x) = x^2$ and that $x > 3$, state the range of f(x).

5 Given that $f(x) = 3x + 4$ and that $f(x) < 0$, state the largest possible integer value of x.

6 $f(x) = 2x + 3$ and $x > 6$

(a) Write down an expression for $f(3x)$

(b) State the domain and range of the function $f(3x)$

7 $f(x) = \frac{x}{3} + 5$, $g(x) = x^2$

(a) Work out the value of fg(4)

(b) Work out an expression for $f^{-1}(x)$

8 $f(x) = 5x + 1$, $g(x) = x^2$

(a) Write down $fg(x)$

(b) Work out the values of x for which $fg(x) = gf(x)$

9 $f(x) = x^2 - 1$

Show that $f(x + 2)$ can be written in the form $(x + a)(x + b)$ where a and b are integers.

A10	**Work with coordinates in all four quadrants**

Teaching Guidance

Students should be able to:

- plot points in all four quadrants
- find and use coordinates of points identified by geometrical information, for example the fourth vertex of a rectangle given the other three vertices
- find coordinates of a midpoint, for example on the diagonal of a rhombus
- identify and use cells in 2D contexts, relating coordinates to applications such as the games Battleships and Connect 4

Notes

Questions may be linked to shapes and other geometrical applications, for example transformations.

Students will be required to identify points with given coordinates and identify coordinates of given points.

Examples

1 Three vertices of rectangle *ABCD* are at *A* (4, 1), *B* (4, –9) and *C* (0, –9)

(a) Work out the coordinates of *D*.

(b) Write down the length and width of the rectangle.

2 *A* is the point (5, 8)

B is the point (9, –12)

(a) Work out the coordinates of the midpoint of *AB*.

(b) Draw a circle with diameter *AB.*

3 *A* is the point (2, 3)

B is the point (–5, 2)

B is the midpoint of *AC*.

Work out the coordinates of *C*.

A11

Plot graphs of equations that correspond to straight-line graphs in the coordinate plane; use the form $y = mx + c$; identify and interpret gradients and intercepts of linear functions graphically and algebraically; understand the gradients of parallel lines

Teaching Guidance

Students should be able to:

- recognise that equations of the form $y = mx + c$ correspond to straight-line graphs in the coordinate plane with gradient m and y-intercept at $(0, c)$.
- draw graphs of functions in which y is given explicitly or implicitly in terms of x
- complete tables of values for straight-line graphs
- calculate the gradient of a given straight-line given two points or from an equation
- manipulate the equations of straight lines so that it is possible to tell whether lines are parallel or not.

Notes

Tables of values may or may not be given.

Examples

1 Draw the graph of $y = 3x - 1$ (with or without a table of values).

2 Draw the graph of $x + 2y = 10$

3 Show clearly that the lines $2x + y = 5$ and $4x = 3 - 2y$ are parallel.

4 $y + 6x = 2$

Work out the gradient and the coordinates of the y-intercept.

5 For a given straight-line graph (such as $y = 3x - 1$ or $x + 2y = 10$), work out the gradient of the line.

6 Work out the gradient of the straight line passing through (2, –6) and (–1, 3)

A11e

Find the equation of the line through two given points, or through one point with a given gradient; understand and use the gradients of perpendicular lines

Teaching Guidance

Students should be able to:

- work out the equation of a line, given two points on the line or given one point and the gradient.
- know that the gradients of perpendicular lines are the negative reciprocal of each other.
- work out the gradients of lines that are perpendicular to a given line
- show that two lines are perpendicular using gradients
- manipulate the equations of straight lines so that it is possible to tell whether or not lines are perpendicular.

Examples

1 *A* has coordinates (3, –5). *B* has coordinates (6, 7)

Work out the equation of the straight line *AB*.

2 Work out the equation of the line that has gradient 3 and passes through (1, –4)

3 Work out the gradient of a line that is perpendicular to the line $2x + 5y = 6$

4 *A* is (2, 3), *B* is (5, 8), *C* is (7, 6) and *D* is (1, –4)

Show that *ABCD* is a trapezium.

A12

Recognise, sketch and interpret graphs of linear functions, quadratic functions, simple cubic functions and the reciprocal function $y = \frac{1}{x}$, with $x \neq 0$

Teaching Guidance

Students should be able to:

- draw, sketch, recognise and interpret the graphs of linear functions
- calculate values for a quadratic and draw the graph
- draw, sketch, recognise and interpret quadratic graphs
- draw, sketch, recognise and interpret graphs of the form $y = x^3 + k$ where k is an integer
- draw, sketch, recognise and interpret the graph $y = \frac{1}{x}$ with $x \neq 0$
- find an approximate value of y for a given value of x, or the approximate values of x for a given value of y.

Notes

Linear graphs should be drawn with a ruler.

Other graphs should be drawn as smooth curves.

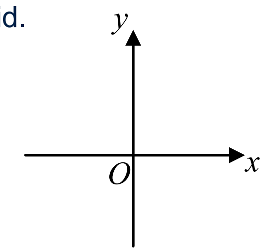
Including using the symmetry of graphs.

Examples

1 Link each function with its sketch.

(Four functions and four sketches given)

2 Sketch the function $y = \frac{1}{x}$ on the grid.



3 (a) Complete a table of values for $y = x^2$ for values of x between –3 and 3

(b) Draw the graph of $y = x^2$

(c) Use the graph to find the approximate values of x when $y = 3$

A12e

Recognise, sketch and interpret graphs of exponential functions $y = k^x$ for positive values of k, and the trigonometrical functions (with arguments in degrees) $y = \sin x$, $y = \cos x$ and $y = \tan x$ for angles of any size

Teaching Guidance

Students should be able to:

- draw, sketch, recognise and interpret graphs of the form $y = k^x$ for positive values of k
- know the shapes of the graphs of the functions $y = \sin x$, $y = \cos x$ and $y = \tan x$

Notes

Students would be expected to sketch a graph of $y = \sin x$, $y = \cos x$ and $y = \tan x$ between 0° and 360°, and know that the maximum and minimum values for sin and cos are 1 and –1

They would also be expected to know that the graphs of sin, cos and tan are periodic.

Graphs should be drawn as smooth curves.

Examples

1 Link each function with its sketch.

(Four functions and four sketches given)

2 (a) Draw a graph of $y = \cos x$ for values of x from 0° to 180°, having completed a table at 30° intervals.

(b) Identify the solutions of $\cos x = -0.5$ between 0° and 360° using symmetry.

3 Apply trigonometric graphs in appropriate context, eg tide heights.

4 Draw the graph of $y = 2^x$ for $0 \leqslant x \leqslant 4$

A13e

Understand and use the gradient function $\frac{dy}{dx}$; differentiation of kx^n where n is a positive integer or 0, and the sum of such functions

Teaching Guidance

Students should be able to:

- understand and use the notation $\frac{dy}{dx}$
- understand the concept of the gradient of a curve
- understand the concept of a rate of change
- find $\frac{dy}{dx}$ when $y = kx^n$, where k is a constant and n is a positive integer or 0, and find $\frac{dy}{dx}$ for the sum of such functions.

Notes

Including expressions which need to be simplified first

Other rules of differentiation will not be required

Questions will not be set which require differentiation from first principles

Examples

1 Work out $\frac{dy}{dx}$

(a) $y = 5x^3 + 6x - 3$ (b) $y = x^2(x + 2)$ (c) $y = (x + 8)(x - 4)$

(d) $y = \frac{x^2 + 7}{4}$ (e) $y = (3x + 2)^2$

2 Work out the value of the gradient function at $x = 2$ when $y = 3x^4 - 2x + 8$

A14e

Know that the gradient of a function is the gradient of the tangent at that point; work out the equation of a tangent at any point on a curve

Teaching Guidance

Students should be able to:

- understand the concept of the gradient of a curve
- work out the gradient of a curve at a point
- work out the equation of a tangent at a point.

Notes

Including expressions which need to be simplified first

Examples

1 Work out the gradient of the curve $y = 4x^2 - 2x + 3$ at the point $(\frac{1}{2}, 3)$.

2 Given that $y = 3x - x^2$

Work out the coordinates of the point at which the gradient of the curve is 5.

3 Work out the equation of the tangent to the curve $y = x^2 - 3x + 5$ at the point (1, 3)

A15e

Use differentiation to find stationary points on a curve: maxima, minima and points of inflection; sketch a curve with known stationary points

Teaching Guidance

Students should be able to

- understand that stationary points are points at which the gradient is zero
- work out stationary points on a curve
- understand the meaning of increasing and decreasing functions
- understand the meaning of maximum points, minimum points and points of inflection
- prove whether a stationary point is a maximum, minimum or point of inflection
- draw a sketch graph of a curve with stationary points.

Notes

It is expected that in questions which ask for a maximum, minimum or inflection point, candidates will prove the nature of the point they have found.

Knowledge of the second derivative is not expected but may be used.

Applications of maxima and minima in real life situations will **not** be set.

Questions may require candidates to work out the values of *x* for which a function is increasing or decreasing.

Students will be expected to sketch curves which have more than one stationary point.

Students will be expected to label the stationary points and where appropriate the intercepts with the *x* and *y* axes.

Students should take care in sketching a stationary point which is a point of inflection.

Examples

1 Prove that the curve $y = x^3 + 3x^2 + 3x - 2$ has only one stationary point.

Show that the stationary point is a point of inflection.

2 Show that the curve $y = 4x - x^4$ has only 1 stationary point.

Determine the nature of this point.

3 Show that $y = \frac{1}{3}x^3 - 3x^2 + 10x - 2$ is an increasing function for all values of x.

4 For what values of x is $y = x^3 - 3x^2 + 5$ a decreasing function?

5 (a) Solve the equation $x^3 + 3x^2 = 0$

(b) Work out the coordinates of the stationary points on the curve $y = x^3 + 3x^2$

(c) Sketch the curve $y = x^3 + 3x^2$

6 (a) Show that the curve $y = 4x - x^4$ has only 1 stationary point.

(b) Sketch the curve $y = 4x - x^4$

A16

Identify and interpret roots, intercepts, and turning points of quadratic functions graphically; deduce roots algebraically

Teaching Guidance

Students should be able to:

- interpret quadratic graphs by finding roots, intercepts and turning points.

Notes

Including the symmetrical property of a quadratic.

Students will be expected to know that the roots of an equation f(x) = 0 can be found where the graph of the function intersects the x-axis.

Examples

1 The graph of $y = x^2 + 2x - 8$ is drawn.

(a) Use the graph to write down the solutions to $x^2 + 2x - 8 = 0$

(b) Work out the coordinates of the turning point.

2 (a) Draw the graph of $y = x^2 + 2x - 1$ for values of x from -3 to 3

(b) Write down the equation of the line of symmetry of the graph.

(c) Work out the coordinates of the turning point.

(d) Using your graph, write down the roots of $x^2 - 2x + 1 = 0$

A16e

Identify and interpret roots, intercepts, turning points of quadratic functions graphically; deduce roots algebraically and turning points by completing the square

Teaching Guidance

Students should be able to:

- complete the square
- deduce turning points by completing the square.

Examples

1 (a) Write $x^2 + 2x - 8$ in the form $(x + a)^2 + b$

(b) Write down the coordinates of the turning point of the curve $y = x^2 + 2x - 8$

2 (a) Write $2x^2 - 12x + 23$ in the form $a(x - b)^2 + c$

(b) Write down the minimum point on the curve $y = 2x^2 - 12x + 23$

A17

Plot and interpret graphs to solve simple kinematic problems involving time, distance, speed and acceleration

Teaching Guidance

Students should be able to:

- plot a graph representing a real-life problem from information given in words, in a table or as a formula
- identify the correct equation of a real-life graph from a drawing of the graph
- read from graphs representing real-life situations; for example, work out the cost of a bill for so many units of gas or the number of units for a given cost, and also understand that the intercept of such a graph represents the fixed charge
- interpret linear graphs representing real-life situations; for example, graphs representing financial situations (eg gas, electricity, water, mobile phone bills, council tax) with or without fixed charges, and also understand that the intercept represents the fixed charge or deposit
- plot and interpret distance-time graphs
- interpret line graphs from real-life situations, for example conversion graphs
- interpret graphs showing real-life situations in geometry, such as the depth of water in containers as they are filled at a steady rate
- interpret non-linear graphs showing real-life situations, such as the height of a ball plotted against time.

Notes

Including problems requiring a graphical solution.

See A12

Examples

1 The cost of hiring a bike is given by the formula $C = 8d + 10$, where d is the number of days for which the bike is hired and C (£) is the total cost of hire.

Draw the graph $C = 8d + 10$

2 For the above graph, what was the deposit required for hiring the bike?

3 Another shop hires out bikes where the cost of hire is given by the formula $C = 5d + 24$

Josh says that the first shop is always cheaper if you want to hire a bike.

Is he correct?

Give reasons for your answer.

4 The cost of hiring a floor-sanding machine is worked out as follows:

Fixed charge = $28

Cost per day = $12

Draw a graph and use it to work out the cost of hiring the machine for six days.

5 Another firm hires out a floor-sanding machine for $22 fixed charge, plus the cost of the first two days at $20 per day, then $8 for each additional day.

Draw a graph on the same axes as the one above to show the cost of hiring the machine for six days.

Which firm would you use to hire the floor-sanding machine for five or more days?

Give reasons for your answer.

6 Draw and interpret a distance-time graph for a car journey.

For how long was the car stopped?

7 Water is being poured at a steady rate into a cylindrical tank.

On given axes, sketch a graph showing depth of water against time taken.

8 5 miles = 8 kilometres.

Draw a suitable graph on the grid provided and use it to convert 43 miles to kilometres.

9 Here is a conversion graph for °C and °F (graph given).

What temperature has the same numerical value in both °C and °F?

10 For this container (image of a container provided), sketch on the grid below the graph of height, h, against time, t, as the water is poured into the container at a constant rate.

11 Four images of different-shaped containers and four different sketches of curves are provided.

Match each container to the correct curve, showing the height of water as the containers are filled at a constant rate.

12 Sketch the graph of volume, V, against time, t, as water flows out of a container at a constant rate.

A17e

Calculate or estimate gradients of graphs and areas under graphs (including quadratic and other non-linear graphs), and interpret results in cases such as distance-time graphs, velocity-time graphs

Teaching Guidance

Students should be able to:

- estimate the gradient at a point on a curve by drawing a tangent at that point and working out its gradient
- interpret the gradient as a rate of change
- interpret the area under the graph
- understand that if the vertical axis represents speed/velocity and the horizontal axis represents time then the gradient will represent acceleration
- understand that if the vertical axis represents distance and the horizontal axis represents time then the gradient will represent speed/velocity
- understand the difference between positive and negative gradients as rates of change
- understand that the rate of change at a particular instant in time is represented by the gradient of the tangent to the curve at that point
- calculate the area under a graph consisting of straight lines
- use the areas of trapezia, triangles and rectangles to estimate the area under a curve.

Notes

The trapezium rule need not be known, but it is recommended as the most efficient means of calculating the area under a curve.

Students should know that the area under a velocity-time graph represents distance.

Students should know that if the vertical axis represents distance on a distance-time graph, then the gradient will represent velocity.

Students should know that if the vertical axis represents velocity on a velocity-time graph, then the gradient will represent acceleration.

Students should understand the difference between positive and negative gradients as increasing speed and decreasing speed on a distance-time graph.

Students should know that the rate of change at a particular instant in time is represented by the gradient of the tangent to the curve at that point.

Velocity will be given as, for example, m/s and acceleration will be given as, for example, m/s^2.

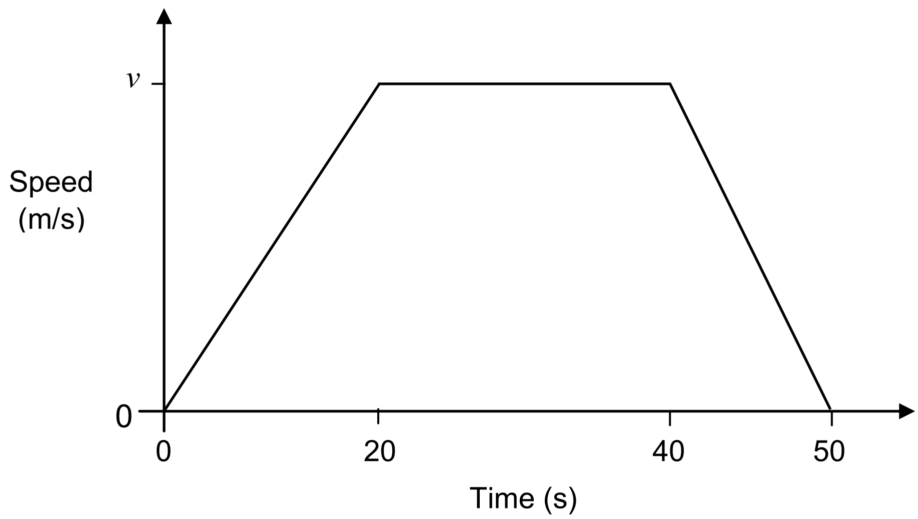
Examples

1 The graph shows the speed of a car between two sets of traffic lights.

It achieves a maximum speed of v metres per second.

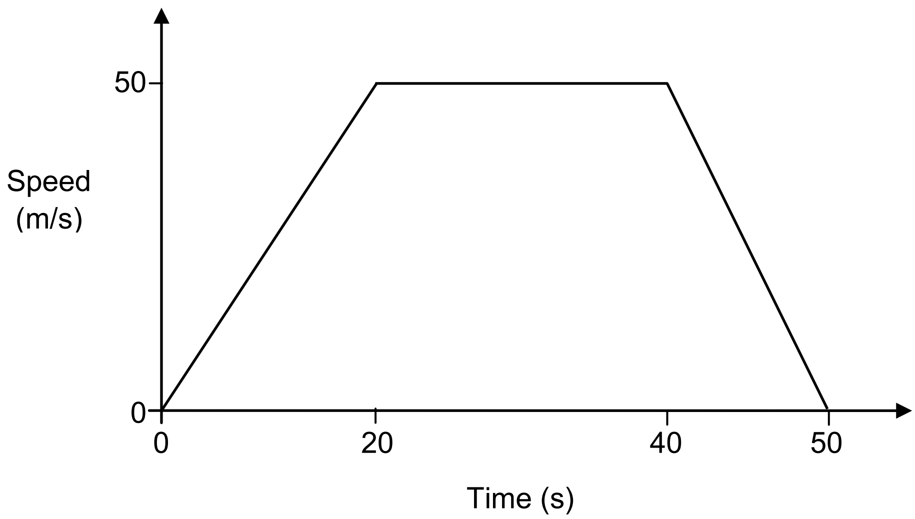
It travels for 50 seconds.

The distance between the traffic lights is 625 metres.



Calculate the value of v.

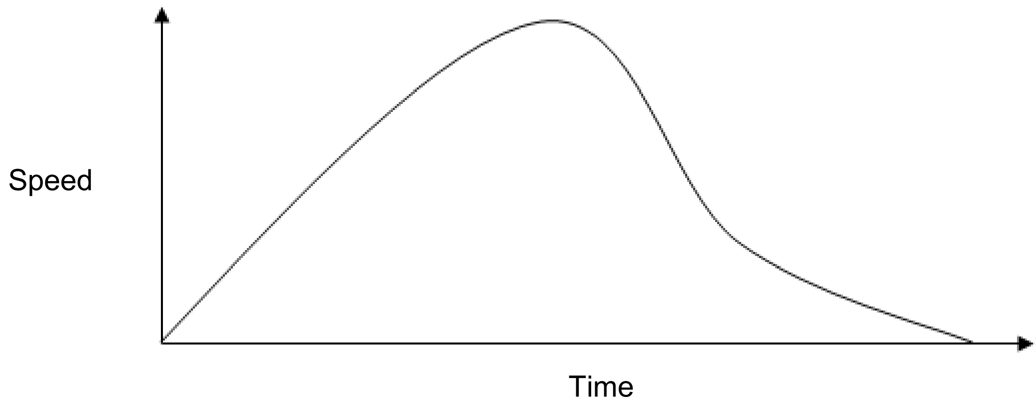
2 The graph shows the speed of a train between two stations.



Calculate the distance between the stations.

3 The graph shows the speed-time graph of a car.

(Drawn on graph paper with axes and values clearly marked)



Use the graph to work out

(a) The maximum speed of the car during this journey.

(b) The total distance travelled.

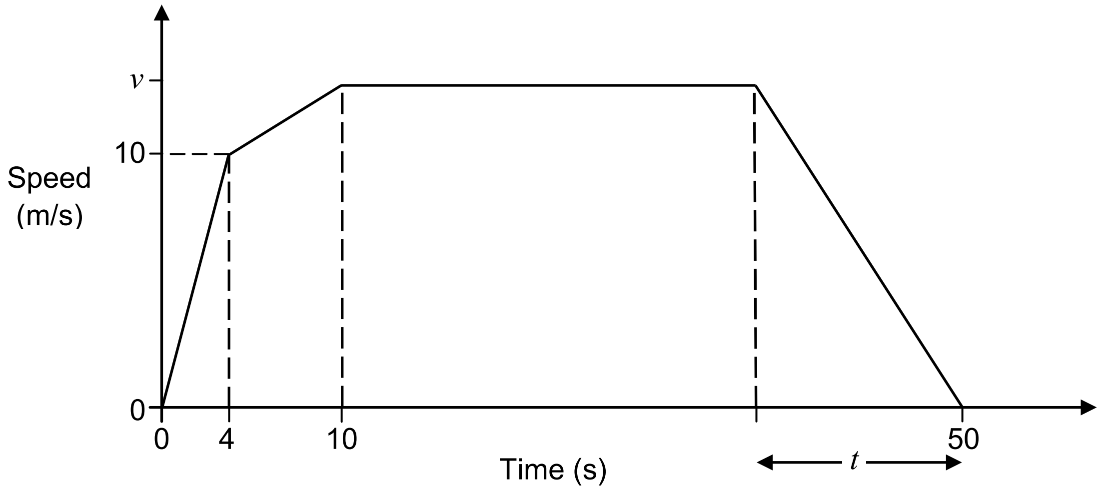
(c) The average speed for the journey.

(d) The deceleration of the car after 8 seconds.

4 The graph shows the speed of a car between two sets of traffic lights.

It achieves a maximum speed of v metres per second.

It travels for 50 seconds.



(a) Calculate the acceleration of the car during the first 4 seconds.

(b) Describe how the motion of the car changes at the end of the tenth second.

(c) The car decelerates for the last t seconds of the motion.

The distance travelled whilst decelerating is 75 metres.

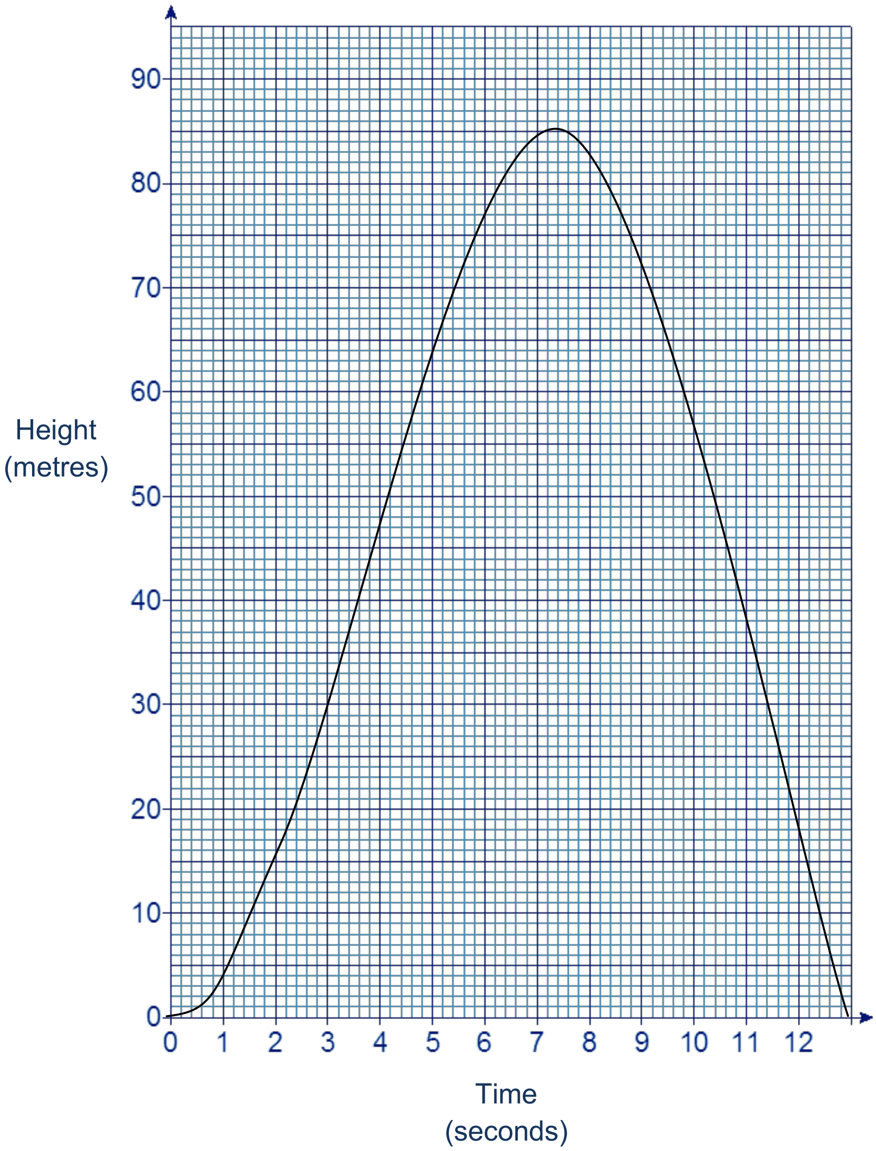
Show that $tv = 150$

(d) The distance travelled at constant speed is 450 metres.

Show that $40v - tv = 450$

(e) Hence, or otherwise, find the total distance between the two sets of traffic lights.

5 The graph shows the height of a firework rocket above the ground plotted against time after take-off.



(a) Use the graph to find the greatest height reached by the rocket.

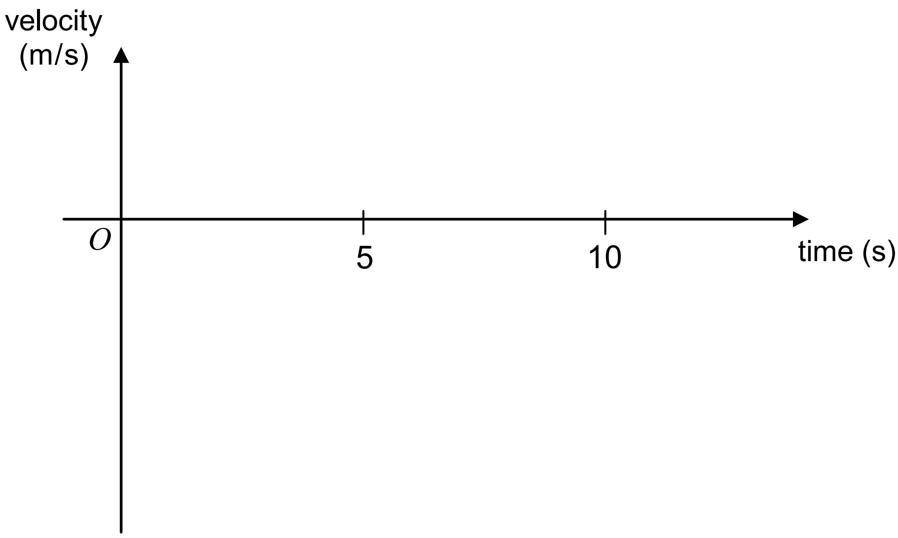
(b) How long is the rocket moving upwards?

(c) When is the rocket rising at its fastest speed?

(d) The rocket accelerates at a constant rate while its chemicals burn.
After this it decelerates at a constant rate.

On the axes below, sketch the velocity-time graph for the rise of the rocket.

(Graph will be drawn full size)



A18e

Express direct and inverse variation in algebraic terms and use this form of expression to find unknown quantities

Teaching Guidance

Students should be able to:

- understand that an equation of the form $y = kx$ represents direct proportion and that k is the constant of proportionality
- understand that an equation of the form $y = \frac{k}{x}$ represents inverse proportion and that k is the constant of proportionality.

Notes

Understand that if l and w are inversely proportional, then $lw = A$, where A is a constant.

Examples

1 Time = $\frac{\text{distance}}{\text{speed}}$

If the distance is doubled and the speed is halved, what happens to the time?

Circle your answer.

$\times\ 2$ $\qquad$ $\times\ 4$ $\qquad$ $\times\ \frac{1}{2}$ $\qquad$ $\times\ \frac{1}{4}$

2 For a rectangle of length l and width w,

$$l = \frac{k}{w}$$

What does the constant k represent?

3 The mass (m) of a sphere is directly proportional to the cube of its radius (r).

When r = 5 cm, m = 500g

Work out the mass of a sphere with r = 10 cm

4 The distance, d kilometres, of the horizon from a point h metres above sea level is given by $d = k\sqrt{h}$

If d = 7.5 when h = 25 find

(a) d when h = 40

(b) h when d = 10

5 The number of beats per minute (b) a pendulum makes is inversely proportional to the square root of its length (l).

A pendulum of length 0.16 m makes 150 beats per minute.

Work out an equation connecting b and l.

6 y is inversely proportional to x^2

When x = 8, y = 2

Work out the value of y when x = 5

6	**Algebra – Solving equations and inequalities**
A19	**Solve linear equations in one unknown algebraically; find approximate solutions using a graph**

Teaching Guidance

Students should be able to:

- solve simple linear equations by using inverse operations or by transforming both sides in the same way
- solve simple linear equations where the unknown appears on one or both sides of the equation or where the equation involves brackets.

Notes

Including use of brackets.

Questions may have solutions that are negative or involve a fraction.

Questions may be set with or without a context.

Examples

1 Solve $4x - 11 = 3$

2 Solve $4x - 1 = x + 4$

3 Solve $5x - 4 = 2(x + 1)$

4 The graph of $y = 8x - 5$ is shown.

Use the graph to write down the solution of $8x - 5 = 0$

Give your answer to 1 decimal place.

A20

Solve quadratic equations algebraically by factorising; find approximate solutions using a graph

Teaching Guidance

Students should be able to:

- solve quadratic equations by factorising
- read approximate solutions from a graph.

Notes

Students should know that trial and improvement is not an acceptable method for solving quadratic equations.

Students should be able to choose or interpret answers to a geometrical problem, for example rejecting a negative solution as a length.

Students should know that the roots of an equation $f(x) = 0$ can be found where the graph of the function intersects the x-axis and that the solution of $f(x) = a$ is found where $y = a$ intersects with $y = f(x)$

Examples

1 Solve $x^2 - 7x + 10 = 0$

2 Solve $x^2 + 3x = 4$

3 The graph of $y = x^2 + x - 3$ is shown.

Write down approximate solutions of $x^2 + x - 3 = 0$

A20e

Solve quadratic equations algebraically by factorising, by completing the square and by using the quadratic formula; find approximate solutions using a graph

Teaching Guidance

Students should be able to:

- solve quadratic equations by factorising, completing the square or using the quadratic formula
- solve geometry problems that lead to a quadratic equation that can be solved by using the quadratic formula
- read approximate solutions from a graph.

Notes

Students should know that trial and improvement is not an acceptable method for solving quadratic equations.

Solutions to quadratic equations, using the quadratic formula or by completing the square, may be left in surd form where appropriate.

Examples

1 Expressions for the side lengths of a rectangle are $2x^2$ cm and $9x$ cm

The perimeter is 10 cm

Work out the area of the rectangle.

Give your answer to a suitable degree of accuracy.

2 Solve $x^2 - 2x - 1 = 0$

Give your answer to 2 decimal places.

3 Write $x^2 + 4x - 9$ in the form $(x + a)^2 - b$

Hence, or otherwise, solve the equation $x^2 + 4x - 9 = 0$, giving answers to 2 decimal places.

4 Solve the equation $\dfrac{4}{2x - 1} - \dfrac{1}{x + 1} = 1$

5 Solve $5x^2 + 13x - 6 = 0$

6 Solve $(x + 4)(x - 6) = 3x$

7 (a) Find the values of a and b such that $x^2 + 6x - 3 \equiv (x + a)^2 + b$

(b) Hence, or otherwise, solve the equation $x^2 + 6x - 3 = 0$

Give your answers in surd form.

8 Solve $\frac{1}{x} + \frac{1}{x+1} = 1$

A21

Solve two linear simultaneous equations in two variables algebraically; find approximate solutions using a graph

Teaching Guidance

Students should be able to:

- solve simultaneous linear equations by elimination or substitution or any other valid method
- find approximate solutions using the point of intersection of two straight lines.

Examples

1 Solve the simultaneous equations

$2x + 3y = 7$

$x + 3y = 2$

You **must** show your working.

Do **not** use trial and improvement.

2 $x + y = 25$

$x - y = 4$

Work out the values of x and y.

A21e

Solve two simultaneous equations with one linear and one quadratic in two variables algebraically; find approximate solutions using a graph

Teaching Guidance

Students should be able to:

- solve simultaneous equations when one is linear and the other quadratic
- appreciate that the solution of f(x) = g(x) is found where y = f(x) intersects with y = g(x)
  eg the points of intersection of the graphs of $y = x^2 + 3x - 10$ and $y = 2x + 1$ are the solutions to the equation $x^2 + 3x - 10 = 2x + 1$ or $x^2 + x - 11 = 0$

Notes

Questions may include geometrical problems, problems set in context and questions requiring a graphical solution.

These may lead to a quadratic equation that can be solved by factorising or by using the quadratic formula, but may also lead to a quadratic equation that can be solved graphically to find approximate solutions.

Examples

1 Solve the simultaneous equations $y = 11x - 2$
$y = 5x^2$

Do **not** use trial and improvement.

You **must** show your working.

2 A straight line has the equation $y = 2x - 3$
A curve has the equation $y^2 = 8x - 16$

Solve these simultaneous equations to find any points of intersection of the line and the curve.

Do **not** use trial and improvement.

You **must** show all your working.

3 The graph of $y = x^2 + 2x - 8$ is drawn.
On the same grid draw the graph of $y = 2x - 3$

(a) Use the graph to write down approximate solutions to $x^2 + 2x - 8 = 2x - 3$

(b) Show that the solutions are $x = \pm\sqrt{5}$

You **must** show your working.

4 The graph of $y = x^2 + 5$ is drawn.

Use the graph to find approximate solutions to $x^2 + 5 = x + 8$

A22 **Translate simple situations or procedures into algebraic expressions or formulae; derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution**

Teaching Guidance

Students should be able to:

- set up simple linear equations
- rearrange simple linear equations
- set up simple linear equations to solve problems
- set up a pair of simultaneous linear equations to solve problems
- interpret solutions of equations in context.

Notes

Questions may include geometrical problems, problems set with or without a context, and questions requiring a graphical solution.

Examples

1 The angles of a triangle (diagram provided) are $2x$, $x + 30$ and $x + 70$

Work out the value of x.

2 The cost of three adult tickets and two child tickets is \$73

The cost of two adult tickets and five child tickets is \$89

Work out the cost of an adult ticket and the cost of a child ticket.

3 The sum of two numbers is 25

The difference between the numbers is 4

Work out the two numbers.

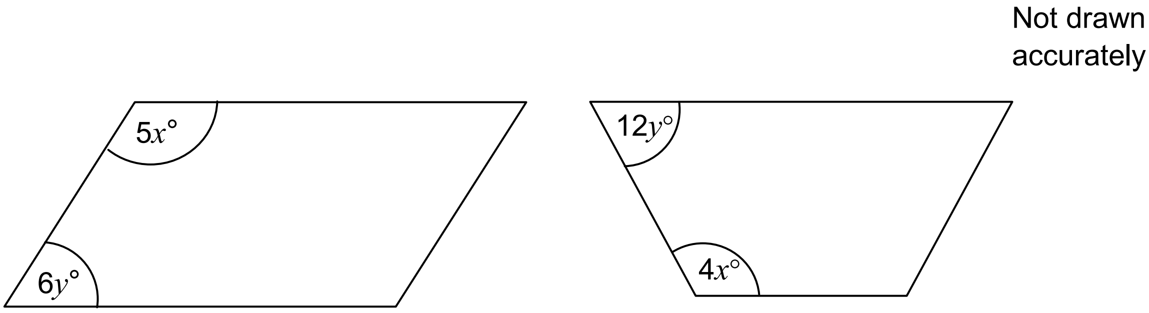
4 Jo is three years older than twice Sam's age.
The sum of their ages is 33

Work out Jo's age.

5 Two of the angles in a parallelogram are x and $(x - 32°)$

Work out the size of the larger angle.

6 The diagram shows a parallelogram and a trapezium.



Work out the values of x and y.

A23

Solve linear inequalities in one variable; represent the solution set on a number line

Teaching Guidance

Students should be able to:

- know the difference between $<$, $\leqslant$, $\geqslant$, $>$ and $\neq$
- solve simple linear inequalities in one variable
- represent the solution set of an inequality on a number line, knowing the correct conventions of an open circle for a strict inequality and a closed circle for an inclusive inequality.

Notes

See N1

Examples

1 Show the inequality $-4 < x \leqslant 2$ on a number line.

2 Solve the inequality $2x - 7 \leqslant 3$ and represent the solution set on a number line.

3 Write down all the integers that satisfy the inequality $-12 \leqslant 3n < 6$

4 Solve $-2x < 18$

A23e

Solve linear inequalities in one or two variables and quadratic inequalities in one variable; represent the solution set on a number line and on a graph

Teaching Guidance

Students should be able to:

- set up inequalities based on the information given in the question
- represent these inequalities on a given coordinate grid
- shade out the side of the boundary line that **does not** satisfy the inequality
- solve quadratic inequalities
- understand and use a solution set of discrete values written in the form $\{ -2, -1, 0, 1, 2 \}$
- understand and use a solution set of continuous values written in the form $-3 < x < 3$

Notes

Questions will define the necessary conditions, which will lead to inequalities such as
$x > 1$, $2x + 3y \leqslant 12$ etc…

Variables may be defined in the question, for example, "Let x represent the number of 50-seater buses."

When drawing boundary lines students are expected to use

- dashed lines for strict inequalities
- solid lines for inclusive inequalities.

It is recommended to shade out the side of a boundary line that does not satisfy the inequality, thus leaving the feasible region blank.

Questions may require factorisation of quadratics. Questions may require solution by completing the square, using the quadratic formula or using a graphical method.

Questions may require integer values that satisfy the inequalities.

Questions may ask for the smallest or largest integer value that satisfies the inequality.

Examples

1 Solve $8(x - 7) < 3(x + 2)$

2 Work out the solution set of integer values for x of the inequality $-4 < 3x + 2 < 17$

3 Solve $5x^2 + 13x - 6 \geqslant 0$

4 Work out the integer values that satisfy the inequality $4x^2 + 7x - 15 < 0$

5 (a) Identify the region represented by $x \leqslant 7$, $y \geqslant 4$ and $y \leqslant x + 3$

(b) What is the maximum value of $x + y$ in this region?

6 The diagram shows the region, P, represented by $x \geqslant 2$, $y \geqslant x$ and $x + y \leqslant 6$
At which vertex of the region is the value of P

(a) a maximum

(b) a minimum

A24

Generate terms of a sequence from either a term-to-term or a position-to-term rule

Teaching Guidance

Students should be able to:

- generate linear sequences
- generate sequences with a given term-to-term rule
- generate simple sequences derived from diagrams and complete a table of results that describes the pattern shown by the diagrams
- describe how a sequence continues
- generate a sequence where the nth term is given
- work out the value of the nth term of a linear sequence for any given value of
- work out the value of the nth term of any sequence for any given value of n.

Notes

Including from patterns and diagrams.

Examples

1 Write down the first three terms of a sequence where the nth term is given by $n^2 + 4$

2 The nth term of a sequence is given by $2n + 5$

(a) Work out the first three terms.

(b) Work out the 100th term.

(c) Which term has a value of 95?

3 The nth term of a sequence is $\dfrac{7 - n}{n^2 + 1}$

(a) Which term in the sequence is the first one with a negative value?

(b) Work out the value of this term.

A25

Recognise and use sequences of triangular, square and cube numbers and simple arithmetic progressions

Teaching Guidance

Students should be able to:

- solve simple problems involving arithmetic progressions.

Examples

1 An arithmetic progression starts $3 + 2a$, $3 + 4a$, $3 + 6a$ …..

If the fifth term is 73, work out the value of a.

A25e **Recognise and use quadratic sequences**

Teaching Guidance

Students should be able to:

- work out the value of the nth term of a quadratic sequence for any given value of n.

Examples

1 The nth term of a sequence is $\frac{1}{2}n(n + 1)$

(a) Work out the 6th term of the sequence.

(b) Which term is the first one with a value greater than 100

2 (a) Write down the next term in the sequence 1, 3, 6, 10, ….

(b) Describe a rule for continuing the sequence.

A26 **Deduce expressions to calculate the nth term of linear sequences**

Teaching Guidance

Students should be able to:

- work out a formula for the nth term of a linear sequence
- work out an expression in terms of n for the nth term of a linear sequence by knowing that the common difference can be used to generate a formula for the nth term.

Examples

1 Write down an expression for the nth term of the following sequence.

3 7 11 15 ....

2 (a) Write down an expression for the nth term of the following sequence.

5 8 11 14 ....

(b) Explain why 61 cannot be a term in this sequence.

A26e

Deduce expressions to calculate the nth term of quadratic sequences

Teaching Guidance

Students should be able to:

- work out a formula for the nth term of a quadratic sequence.

Examples

1 Work out the formula for the nth term of the quadratic sequence 5 11 19 29 …..

2 A quadratic sequence starts 3 8 15 24 …..

(a) Show that the nth term is $n^2 + 2n$

(b) Hence find the term that has value 440

G1

Use conventional terms and notations: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons; use the standard conventions for labelling and referring to the sides and angles of triangles; draw diagrams from written description

Teaching Guidance

Students should be able to:

- distinguish between acute, obtuse, reflex and right angles
- name a type of angle
- use one lower-case letter or three upper-case letters to represent an angle, for example *x* or *ABC*
- understand and draw lines that are parallel
- understand that two lines that are perpendicular are at 90° to each other
- identify lines that are perpendicular
- draw a perpendicular line in a diagram
- use geometrical language
- use letters to identify points and lines
- recognise that, for example, in a rectangle *ABCD* the points *A*, *B*, *C* and *D* go around in order.

Examples

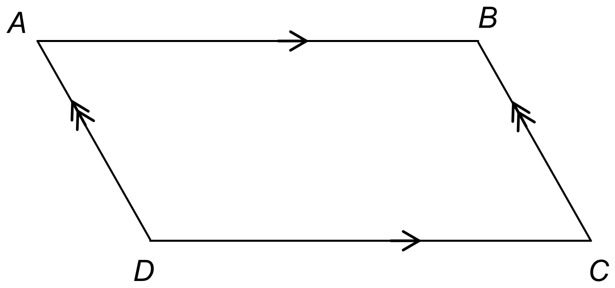
1 Three angles shown.

Three types of angles given.

Students have to match them.

2 Given a line, students need to draw a line perpendicular to this.

3 In parallelogram *ABCD*,



(a) Draw a line through *B* perpendicular to *DC*.

(b) Draw a line through the midpoint of *AB* parallel to *CB*.

G2

Recall and use properties of angles at a point, angles at a point on a straight line, vertically opposite angles;

Teaching Guidance

Students should be able to:

- work out the size of missing angles at a point
- work out the size of missing angles at a point on a straight line
- know that vertically opposite angles are equal
- justify an answer with explanations such as ‘angles on a straight line’ etc.

Examples

1 Three angles form a straight line.

Two of the angles are equal.

One of the angles is 30° more than another angle.

Work out **two** possible sets of values for the three angles.

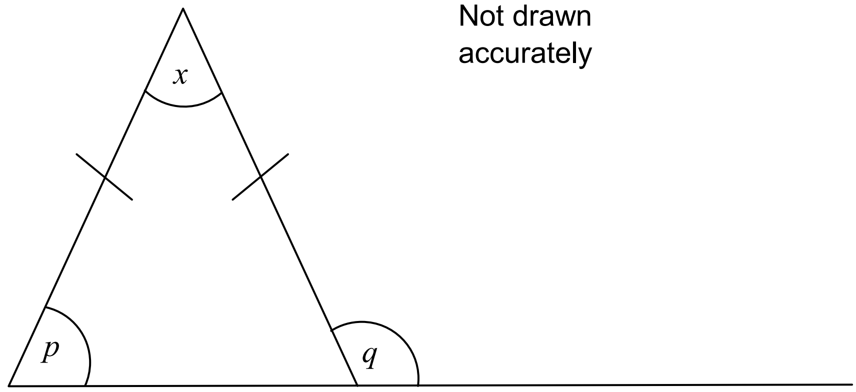
2 There are three angles at a point.

One is acute, one is obtuse and one is reflex.

Write down **one** possible set of three angles.

3 Given two intersecting lines with angles x and $4x$ at the vertex, work out the larger angle.

2 The diagram shows an isosceles triangle on a straight line.



(a) If $x = 30°$, work out the value of p.

(b) If $q = 100°$, work out the value of x.

(c) Write down the value of q in terms of x and p.

G3

Understand and use the angle properties of parallel and intersecting lines, triangles and quadrilaterals

Teaching Guidance

Students should be able to:

- understand and use the angle properties of parallel lines
- recall and use the terms 'alternate angles' and 'corresponding angles'
- work out missing angles using properties of alternate angles, corresponding angles and interior angles
- understand the consequent properties of parallelograms
- understand the proof that the angle sum of a triangle is 180°
- understand the proof that the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices
- use angle properties of equilateral, isosceles and right-angled triangles
- use the fact that the angle sum of a quadrilateral is 360°

Notes

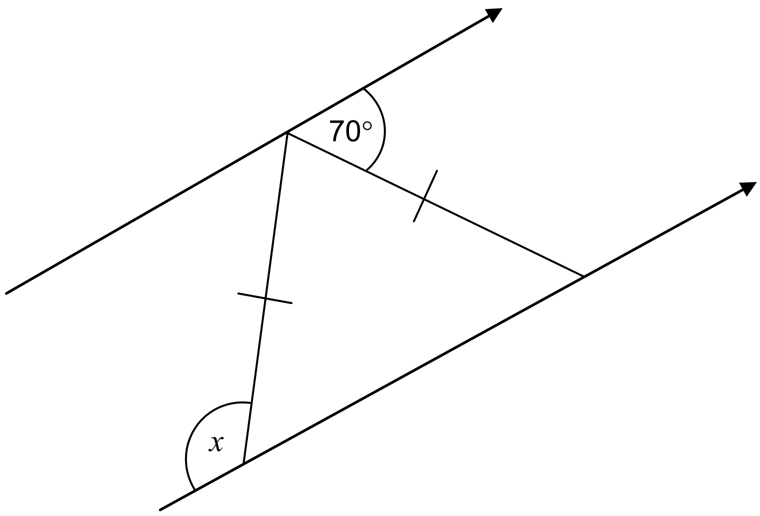
Colloquial terms such as *Z* angles are not acceptable and should not be used.

Students should know the names and properties of isosceles, equilateral and scalene triangles, and also right-angled, acute-angled and obtuse-angled triangles.

Examples

1 Work out the size of angle x.

You must explain any properties that you have used to obtain your answer.



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2 One of the angles in an isosceles triangle is 64°

Work out the size of the smallest possible angle in the triangle.

G4	**Calculate and use the sums of the interior and exterior angles of polygons**

Teaching Guidance

Students should be able to:

- calculate and use the sums of interior angles of polygons
- recognise and name regular polygons, pentagons, hexagons, octagons and decagons
- use the angle sum of irregular polygons
- calculate and use the angles of regular polygons
- use the fact that the sum of the interior angles of an n-sided polygon is $180(n - 2)$
- use the fact that the sum of the exterior angles of any polygon is 360°
- use the relationship interior angle + exterior angle = 180°
- use the sum of the interior angles of a triangle to deduce the sum of the interior angles of any polygon.

Notes

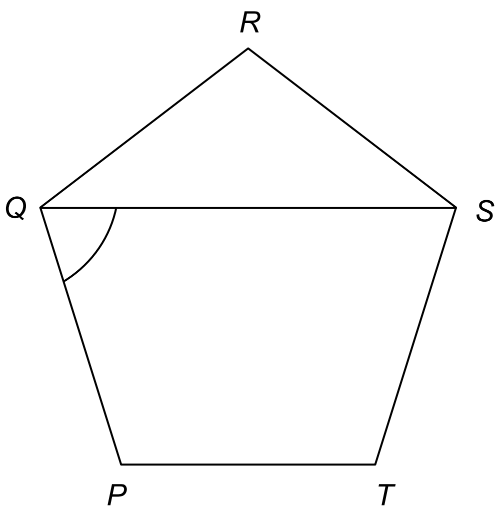
Students should be able to calculate the values of the interior angle, exterior angle and angle at the centre of regular polygons

Examples

1 The pentagon *PQRST* has sides of equal length.

The line *QS* is drawn.

Not drawn accurately



Work out the size of angle *PQS*.

2 Work out the size of each interior angle of a regular hexagon.

3 Explain why angles of 99° and 91° do not fit together to make a straight line.

4 The exterior angle of a regular polygon has size 30°

How many sides does the polygon have?

G5

Recall the properties and definitions of special types of quadrilaterals, including square, rectangle, parallelogram, trapezium, kite and rhombus

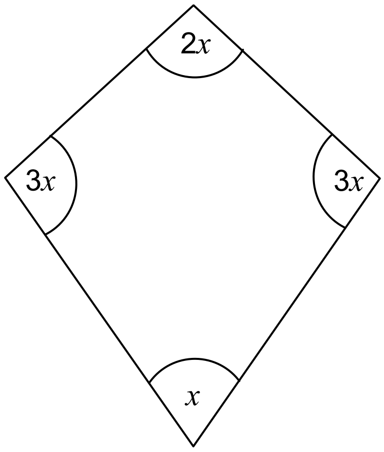
Teaching Guidance

Students should be able to:

- recall the properties and definitions of special types of quadrilaterals
- name a given shape
- identify and use symmetries of special types of quadrilaterals
- identify a shape given its properties
- list the properties of a given shape
- draw a sketch of a named shape
- identify quadrilaterals that have common properties
- classify quadrilaterals using common geometric properties.

Examples

1 In this quadrilateral the angles are x, $2x$, $3x$ and $3x$ as shown.

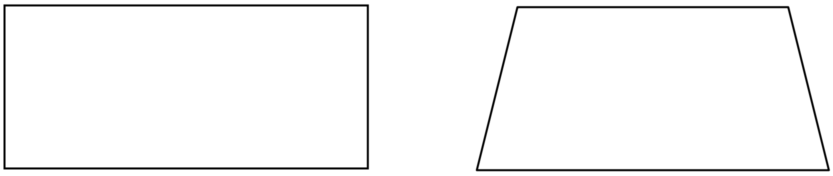


Not drawn accurately

What name is given to this shape?

Show that the shape has two acute and two obtuse angles.

2 Write down two similarities and two differences between a rectangle and a trapezium.



3 Ben is describing a shape.

- It has four sides
- The sides are all the same length
- It is not a square.

(a) What shape is Ben describing?

(b) Write down another fact about this shape.

G6	**Recognise reflection and rotation symmetry of 2D shapes**

Teaching Guidance

Students should be able to:

- recognise reflection symmetry of 2D shapes
- understand line symmetry
- identify lines of symmetry on a shape or diagram
- draw lines of symmetry on a shape or diagram
- draw or complete a diagram with a given number of lines of symmetry
- recognise rotational symmetry of 2D shapes
- identify the order of rotational symmetry on a shape or diagram
- draw or complete a diagram with rotational symmetry
- identify and draw lines of symmetry on a Cartesian grid
- identify the order of rotational symmetry of shapes on a Cartesian grid
- draw or complete a diagram with rotational symmetry on a Cartesian grid.

Notes

Including knowing the symmetry properties of regular polygons

Examples

1 A shape has three lines of symmetry.
All sides are the same length.

Write down the name of the shape.

2 Draw a shape with two lines of symmetry and rotational symmetry of order 2

3 Describe all the symmetries of a shape.
(Diagram given)

4 Shade in squares on a grid so that 75% of the squares are shaded and the shaded shape has line symmetry.

G7 **Understand congruence and similarity; calculate lengths of similar figures**

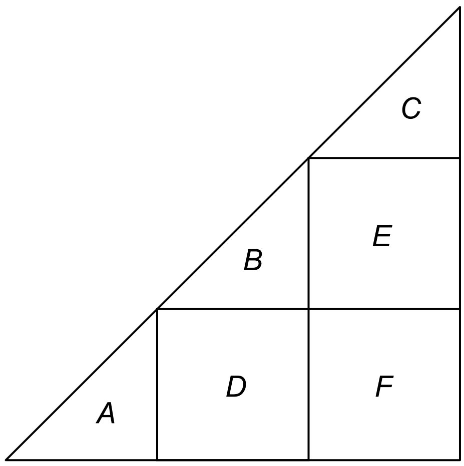
Teaching Guidance

Students should be able to:

- understand congruence
- identify shapes that are congruent
- recognise congruent shapes when rotated, reflected or in different orientations
- understand the effect of enlargement on perimeter
- work out the side of one shape that is similar to another shape given the ratio or scale factor of lengths.

Examples

1 This diagram is made up of triangles and squares as shown.



(a) Write down a letter for a triangle that is congruent to triangle *C*.

(b) Use some of the letters to write down a triangle that is similar to the triangle made up of *B*, *C* and *E*.

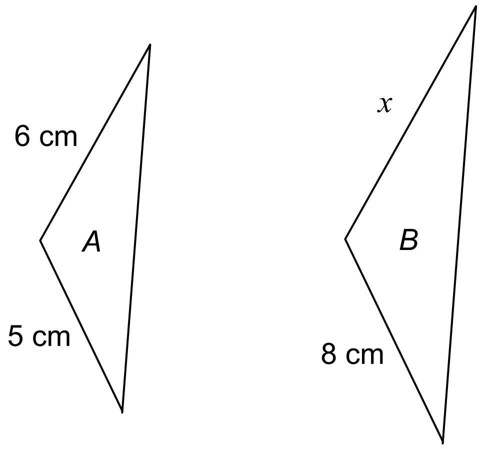
2 Given several shapes, identify pairs of congruent shapes.

3 Given several shapes, identify the odd ones out, giving reasons.

4 *A* and *B* are similar triangles.

Work out x.

Not drawn accurately



G7e

Understand and use conditions for congruent triangles

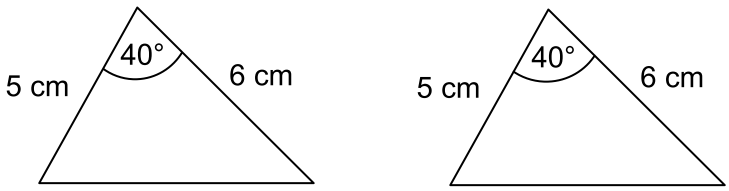
Teaching Guidance

Students should be able to:

- understand and use conditions for congruent triangles: SSS, SAS, ASA and RHS
- understand and use SSS, SAS, ASA and RHS conditions to prove the congruence of triangles using formal arguments.

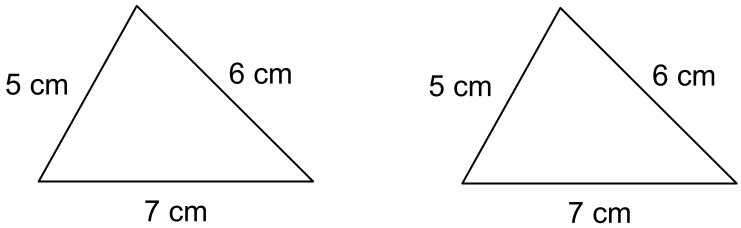
Examples

1 Circle the reason why each pair of triangles are congruent.



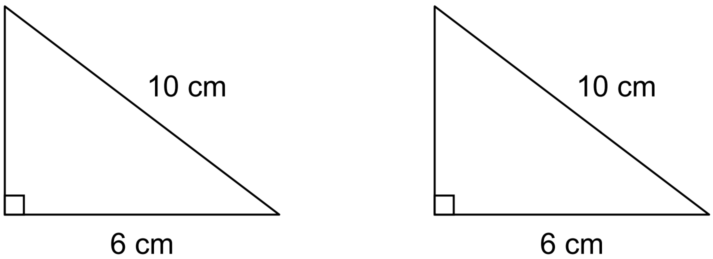
SSS ASA

SAS RHS



SSS ASA

SAS RHS



SSS ASA

SAS RHS

G8	**Identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment**

Teaching Guidance

Students should be able to:

- recall the definition of a circle
- identify and name the parts of a circle
- draw the parts of a circle
- understand related terms of a circle
- draw a circle given the radius or diameter.

Examples

1 Draw a chord on a given circle.

2 Chords of a circle are joined together inside the circle to make a regular shape.
The length of each chord is equal to the radius of the circle.

How many chords are joined together?

3 Draw a chord perpendicular to a given diameter.

G8e	**Apply the standard circle theorems concerning angles, radii, tangents and chords, and use them to prove related results**

Teaching Guidance

Students should be able to:

- understand that the tangent at any point on a circle is perpendicular to the radius at that point
- understand and use the fact that tangents from an external point are equal in length
- use congruent triangles to explain why the perpendicular from the centre to a chord bisects the chord
- understand that inscribed regular polygons can be constructed by equal division of a circle
- use the fact that the angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the circumference
- use the fact that the angle subtended at the circumference by a semicircle is a right angle
- use the fact that angles in the same segment are equal
- use the fact that opposite angles of a cyclic quadrilateral sum to 180°
- use the alternate segment theorem.

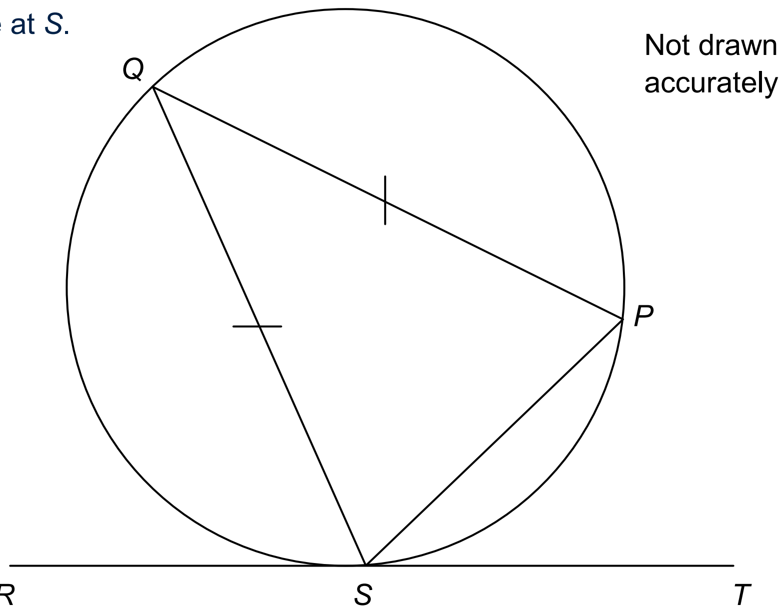
Notes

In formal proofs, it is expected that clear and logical steps are shown with reasons given.

Examples

1 *RST* is a tangent to the circle at *S*.

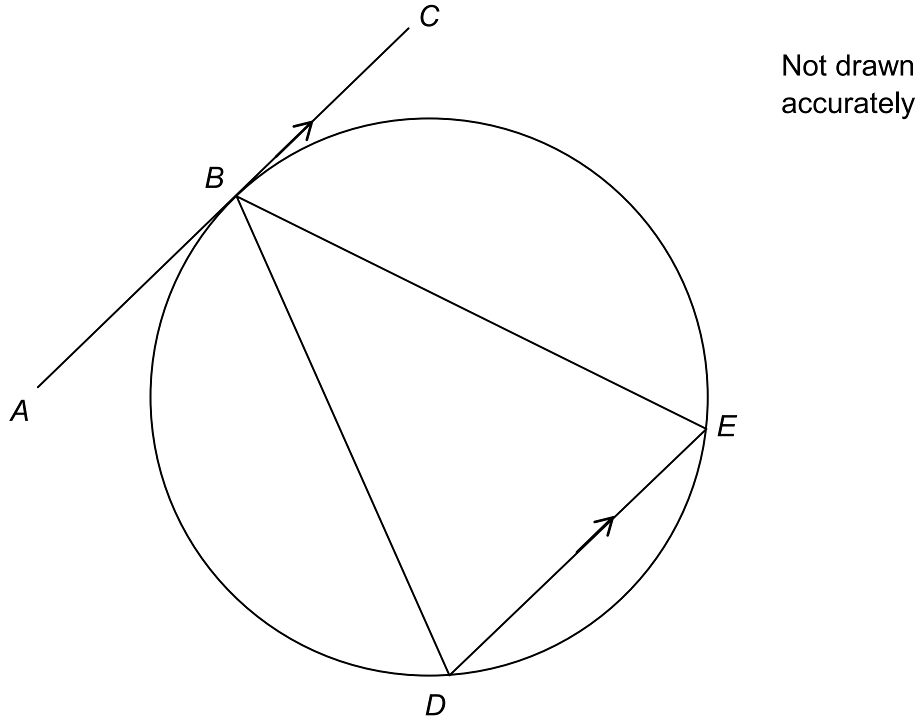
PQ = *QS*



Prove that *QS* bisects angle *RSP*.

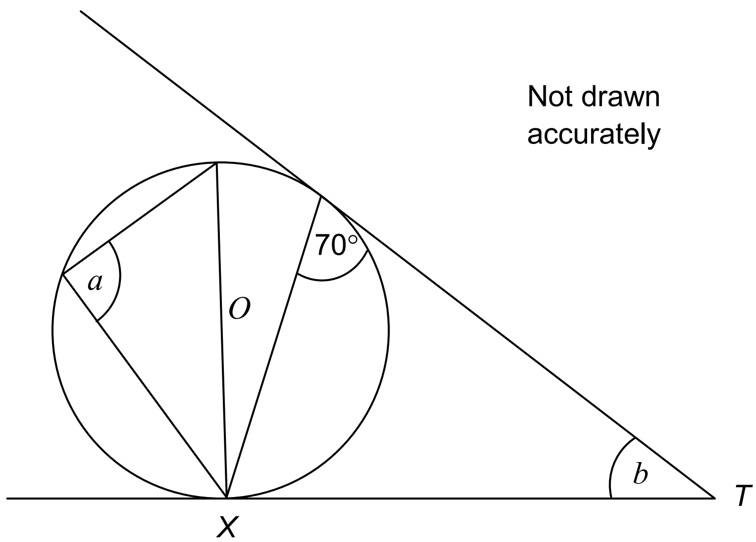
2 *ABC* is a tangent to the circle at *B*.

AC is parallel to *DE*.



Prove that triangle *BDE* is isoceles.

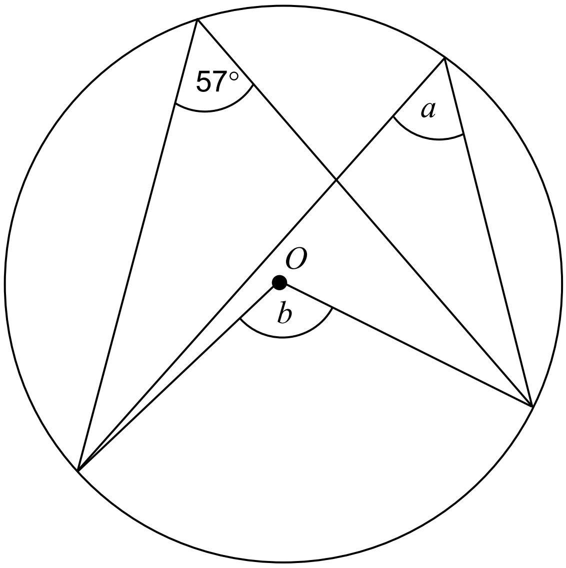
3 *O* is the centre of the circle.



Work out the sizes of angles a and b.

4 O is the centre of the circle.

Not drawn accurately



Work out the sizes of angles a and b.

G9e

Geometrical reasoning and proof: use standard theorems to justify results in geometric contexts

Teaching Guidance

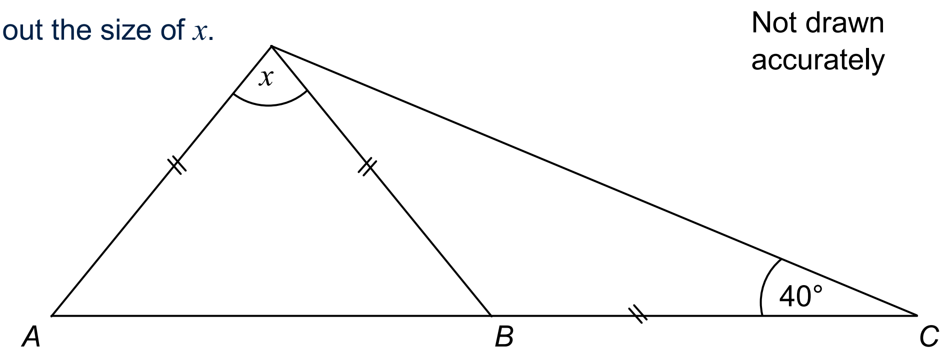
Students should be able to:

- apply mathematical reasoning, explaining and justifying inferences and deductions
- show step-by-step deduction in solving a geometrical problem
- state constraints and give starting points when making deductions.

Examples

1 *ABC* is a straight line.

Work out the size of x.



2 Given that one angle in an isosceles triangle is 70°, show that there are two possible solutions for the other angles.

3 Given that one angle in an isosceles triangle is 90°, show that there can only be one solution for the other angles.

G10 | **Identify properties of the faces, surfaces, edges and vertices of: cubes, cuboids, prisms, cylinders, pyramids, cones and spheres**

Teaching Guidance

Students should be able to:

- know the terms face, edge and vertex (vertices)
- identify and name common solids, for example cube, cuboid, prism, cylinder, pyramid, cone and sphere
- understand that cubes, cuboids, prisms and cylinders have uniform areas of cross-section.

Examples

1 Work out the total number of faces on a triangular prism, a cuboid and a square-based pyramid.

2 Here is the cross-section of a prism (diagram given).

How many faces does the prism have?

G11 **Interpret plans and elevations of 3D shapes; construct and interpret plans and elevations of 3D shapes**

Teaching Guidance

Students should be able to:

- use 2D representations of 3D shapes
- draw nets and show how they fold to make a 3D solid
- analyse 3D shapes through 2D projections and cross sections, including plans and elevations
- understand and draw front and side elevations and plans of shapes made from simple solids, for example a solid made from small cubes
- understand and use isometric drawings.

Examples

1 Identify possible nets for a cube from several drawings.

2 Use plan, front and side elevation drawings to work out the volume of a simple shape.

3 From an isometric drawing, work out the surface area of a cuboid.

G12

Measure line segments and angles in geometric figures, including interpreting maps and scale drawings and use of bearings

Teaching Guidance

Students should be able to:

- use and interpret maps and scale drawings
- use a scale to work out a length on a map or a real length from a map
- use a scale with an actual length to work out a length on a map
- construct scale drawings
- use scale to estimate a length, for example use the height of a man to estimate the height of a building where both are shown in a scale drawing
- work out a scale from a scale drawing given additional information
- use bearings to specify direction
- recall and use the eight points of the compass (N, NE, E, SE, S, SW, W, NW) and their equivalent three-figure bearings
- use three-figure bearings to specify direction
- mark points on a diagram given the bearing from another point
- draw a bearing between points on a map or scale drawing
- measure the bearing of a point from another given point
- work out the bearing of a point from another given point
- work out the bearing to return to a point, given the bearing to leave that point.

Notes

Scale could be given as a ratio (for example 1 : 500000) or as a key (for example 1 cm represents 5 km).

Examples

1 Given the road distance between two ports, use a scale drawing to compare the time taken to travel by car or by boat.

2 Use a scale of 1 : 500 000 to decide how many kilometres are represented by 3 cm on a map.

3 Use accurate constructions to locate a point on a map or scale drawing.

4 Write down the three-figure bearing for NW.

5 Work out the angle between North East and South.

6 Given the bearing of *B* from *A*, work out the bearing of *A* from *B*.

G13

Use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle, constructing an angle of 60°); use these to construct given figures and solve loci problems; know that the perpendicular distance from a point to a line is the shortest distance to the line

Teaching Guidance

Students should be able to:

- measure and draw lines to the nearest mm
- measure and draw angles to the nearest degree
- make accurate drawings of triangles and other 2D shapes using a ruler and a protractor
- make an accurate scale drawing from a sketch, diagram or description
- use a straight edge and a pair of compasses to do standard constructions
- construct a triangle
- construct an equilateral triangle with a given side or given side length
- construct a perpendicular bisector of a given line
- construct a perpendicular at a given point on a given line
- construct a perpendicular from a given point to a given line
- construct an angle bisector
- construct an angle of 60°
- draw parallel lines
- draw circles or part circles given the radius or diameter
- construct diagrams of 2D shapes
- construct a region, for example, bounded by a circle and an intersecting line
- construct loci, for example, given a fixed distance from a point and a fixed distance from a given line
- construct loci, for example, given equal distances from two points
- construct loci, for example, given equal distances from two line segments
- construct a region that is defined as, for example, less than a given distance or greater than a given distance from a point or line segment
- describe regions satisfying several conditions.

Notes

Students will be expected to show clear evidence that a straight edge and compasses have been used to do constructions.

Loci problems may be set in practical contexts such as finding the position of a radio transmitter.

Examples

1 Construct the perpendicular bisector of a line and use this to draw an isosceles triangle.

2 Find the overlapping area of two transmitters, with ranges of 30 km and 40 km respectively.

3 In the diagram *P*, *Q* and *R* represent three trees. Shade the region of points that are less than 30 metres from *P*, less than 40 metres from *Q* and that are nearer to *Q* than *R*.

4 Given a scale drawing of a garden, draw on the diagram the position of a circular pond of radius 0.8 metres which has to be at least 2 metres from any boundary wall.

5 Draw a line parallel to a given line at a distance 3 cm apart.

6 Draw a semicircle of radius 5 cm

7 Construct a triangle with sides of 6 cm, 7 cm and 8 cm

8 Construct a rectangle with sides of 6 cm and 4 cm

9 Using ruler and compasses only, construct a triangle *ABC* such that *AB* = 8 cm, angle *A* = 60° and angle *B* = 45°

G14

Use standard units of measure and related concepts (length, area, volume/ capacity, mass, time, money etc); change freely between related standard units (eg time, length, area, volume/capacity, mass) and compound units (eg speed and density)

Teaching Guidance

Students should be able to:

- interpret scales on a range of measuring instruments, including those for time, temperature and mass, reading from the scale or marking a point on a scale to show a stated value
- know that measurements using real numbers depend on the choice of unit
- recognise that measurements given to the nearest whole unit may be inaccurate by up to one half in either direction
- make sensible estimates of a range of measures in real-life situations, for example estimate the height of a man
- choose appropriate units for estimating measurements, for example the height of a television mast would be measured in metres
- convert between metric measures
- recall and use conversions for metric measures for length, area, volume and capacity
- use conversions between imperial units and metric units and vice versa using common approximations, for example 5 miles $\approx$ 8 kilometres, 1 gallon $\approx$ 4.5 litres, 2.2 pounds $\approx$ 1 kilogram, 1 inch $\approx$ 2.5 centimetres (conversions will always be given in the question)
- know and use the relationship between density, mass and volume.

Examples

1 Read a temperature scale.

2 Mark a given value on a weighing scale.

3 Given a scale with a maximum measurement of 2 kg, explain how 5 kg could be weighed out using the scale.

5 Use the height of a man to estimate the height of a bridge.

6 Estimate the height of a building and use this to estimate the number of pieces of drainpipe of a given length needed to be placed from the top to the bottom of the building.

7 Use 5 mph = 8 km/h to convert 45 mph into km/h

8 Use 5 miles = 8 kilometres

Emma is on holiday in France and agrees to meet her friend half way along the road between their hotels.

Emma's car measures distances in miles.

The distance between the hotels is 32 km

How many miles is it to the meeting point?

9 Use 1 pound = 16 ounces and 2.2 pounds = 1 kilogram

A recipe needs 200 grams of flour.

How many ounces of flour are needed?

10 20 cm^3 of copper has a mass of 179.2 grams.

Work out the density of copper.

11 The density of steel is 1050 kg/m^3

A block of steel is a cuboid measuring 3 m by 2 m by 1.5 m

Work out the mass of the block.

G15

Know and apply formulae to calculate: area of triangles, parallelograms, trapezia; volumes of 3D shapes using $V = Ah$ where A is the constant cross-sectional area and h is the height/length

Teaching Guidance

Students should be able to:

- recall and use the formulae for the area of a rectangle, triangle, parallelogram and trapezium
- work out the area of a rectangle
- work out the area of a triangle
- work out the area of a parallelogram
- work out the area of a trapezium
- calculate the area of compound shapes made from triangles and rectangles
- calculate the area of compound shapes made from two or more rectangles, for example an L shape or T shape
- calculate the area of shapes drawn on a grid, including by counting squares
- calculate the area of simple shapes
- work out the surface area of nets made up of rectangles and triangles
- recall and use the formula for the volume of a cube or cuboid
- recall and use the formula for the volume of a cylinder
- recall and use the formula for the volume of a prism
- work out the volume of a cube or cuboid
- work out the volume of a cylinder
- work out the volume of a prism, for example a triangular prism.

Notes

Students may be required to measure lengths in order to work out areas.

Examples

1 The area of a triangle = 24 cm^2

The base of the triangle is 8 cm

Work out the height of the triangle.

2 The perimeter of a rectangle is 30 cm

The length of the rectangle is double the width.

Work out the area of the rectangle.

3 The area of the base of a cylinder is 20 cm^2

The height of the cylinder is 7 cm

Work out the volume of the cylinder.

State the units of your answer.

4 A cuboid has the same volume as a cube with edges of length 8 cm

(a) Work out one set of possible values for the length, width and height of the cuboid if all three lengths are different.

(b) Work out one set of possible values for the length, width and height of the cuboid if two of the lengths are the same.

5 The volume of a cuboid is 36 cm^3

The area of one of the faces is 9 cm^2

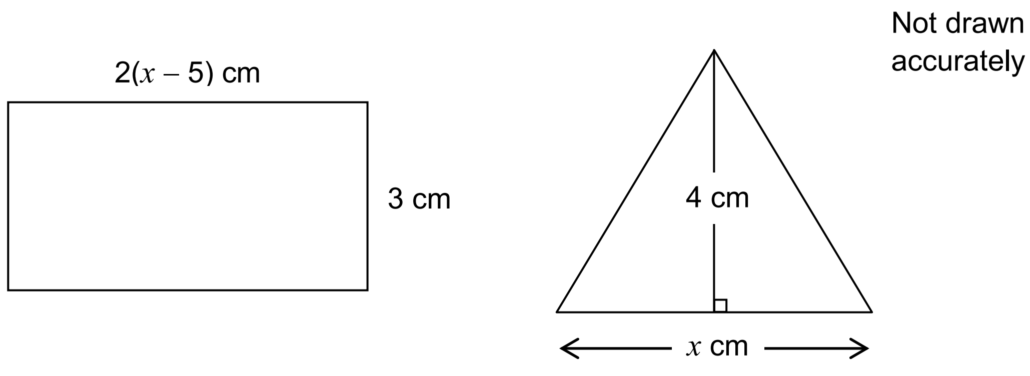
All edges are a whole number of centimetres long.

The length, width and height are all different.

Work out the three dimensions of the cuboid.

6 The area of the rectangle is equal to the area of the triangle.

Work out the value of x.



G16

Know and use the formulae: circumference of a circle= $2\pi r = \pi d$, area of a circle = πr^2; calculate perimeters and areas of 2D shapes, including composite shapes

Teaching Guidance

Students should be able to:

- work out the perimeter of a rectangle
- work out the perimeter of a triangle
- calculate the perimeter of compound shapes made from triangles and rectangles
- calculate the perimeter of compound shapes made from two or more rectangles
- calculate the perimeter of shapes drawn on a grid
- calculate the perimeter of simple shapes
- recall and use the formula for the circumference of a circle
- work out the circumference of a circle given the radius or diameter
- work out the radius or diameter of a circle given the circumference
- use π = 3.142 or the π button on a calculator
- work out the perimeter of semicircles, quarter circles or other fractions of a circle
- recall and use the formula for the area of a circle
- work out the area of a circle, given the radius or diameter
- work out the radius or diameter of a circle given the area
- work out the area of semicircles, quarter circles or other fractions of a circle.

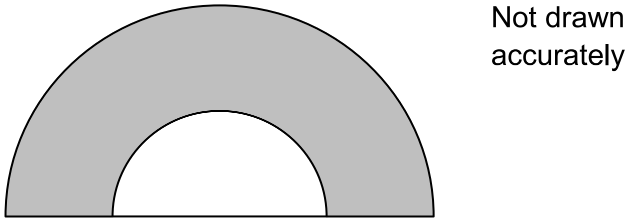
Notes

Students may be required to measure lengths in order to work out perimeters or areas.

For problems involving circles, the answer may be asked for as a multiple of π.

Examples

1 The following diagram shows two semicircles of radius 5 cm and 10 cm



Work out the shaded area.

2 The circumference of a circle of radius 4 cm is equal to the perimeter of a square.

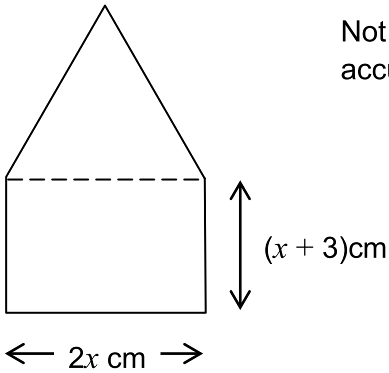
Work out the length of one side of the square.

3 Which is greater, the area of a quarter-circle of radius 10 cm or the area of a semicircle of radius 5 cm?

Show how you decide.

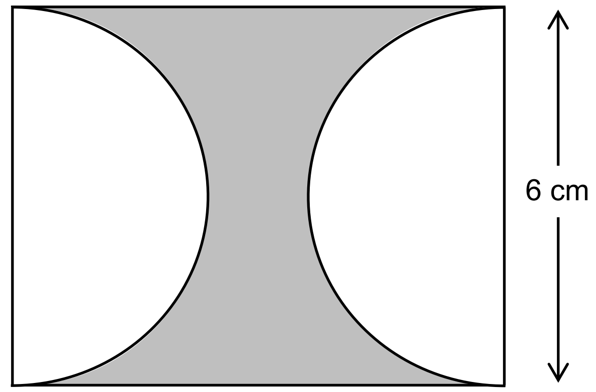
4 A shape is made by joining an equilateral triangle to a rectangle.

The perimeter of the shape is 90 cm



Work out the value of x.

5 A rectangle has two semicircles drawn inside it as shown.



Show that the shaded area is $9(5 - \pi)$ cm^2

G16e	**Know and use the formulae for surface area and volume of spheres, pyramids, cones, including composite shapes and frustums of pyramids and cones**

Teaching Guidance

Students should be able to:

- work out the surface area of spheres, pyramids and cones
- work out the surface area of compound solids constructed from cubes, cuboids, cones, pyramids, cylinders, spheres and hemispheres
- work out volume of spheres, pyramids and cones
- work out the volume of compound solids constructed from cubes, cuboids, cones, pyramids, cylinders, spheres and hemispheres
- solve real-life problems using known solid shapes.

Notes

For problems involving circles, the answer may be asked for as a multiple of π.

The formulae for volume and surface area of a sphere, cone and pyramid will be given in the relevant question.

Examples

1 Work out the total surface area of a cone with diameter 12 cm and slant height 18 cm

2 Work out the volume of a cylinder joined to a hemisphere of the same radius (diagram given).

G17e

Use the relationships between lengths, areas and volumes in similar figures

Teaching Guidance

Students should be able to:

- understand the effect of enlargement on areas of shapes
- understand the effect of enlargement on the surface areas and volumes of solids
- compare the areas or volumes of similar shapes or solids, knowing that if $a : b$ is the ratio of lengths, then $a^2 : b^2$ is the ratio of areas and $a^3 : b^3$ is the ratio of volumes
- work out the area or volume of one shape/solid given the area or volume of a similar shape/solid and the ratio or scale factor of lengths of the shape/solid.

Notes

Questions may be set which ask, for example, how many times bigger is the area of shape *A* than shape *B*?

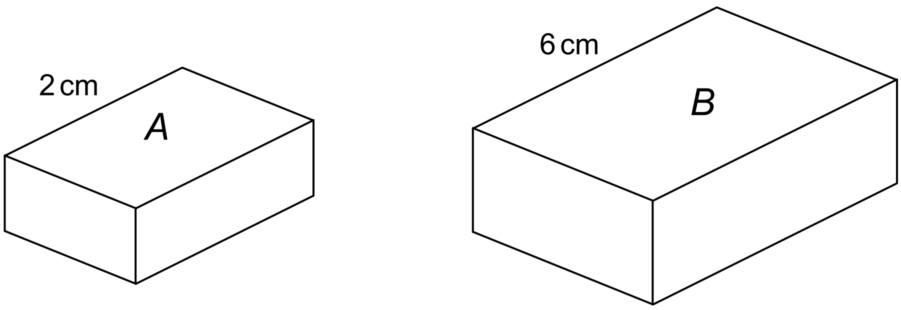
Students will be expected to know the connection between the linear, area and volume scale factors of similar shapes and solids.

Questions may be asked which involve the relationship between weight and volume, area and cost of paint, etc.

Scales will be given as, for example, 1 cm represents 1 km or 1 : 100

Examples

1 These boxes are similar.



What is the ratio of the volumes of box *A* to box *B*?

2 Two statues are similar in shape.

The scale factor of the enlargement is 1.5

The smaller statue has mass of 0.8 tonnes.

Work out the mass of the larger statue.

3 What is the ratio of the surface areas of two similar cones with base radii 3 cm and 12 cm respectively?

4 Two solids are similar.

The ratio of their lengths is 1 : 2

Write down the volume of the small solid as a fraction of the volume of the large solid.

G18e

Calculate arc lengths, angles and areas of sectors of circles

Teaching Guidance

Students should be able to:

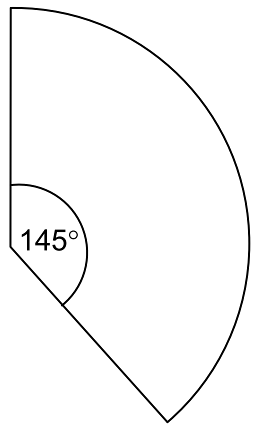
- calculate the length of arcs of circles
- calculate the area of sectors of circles
- given the length of an arc or area of a secto, calculate the angle subtended at the centre or the length of the radius.

Examples

1 The diagram shows a sector of a circle.

The arc length is 24 cm

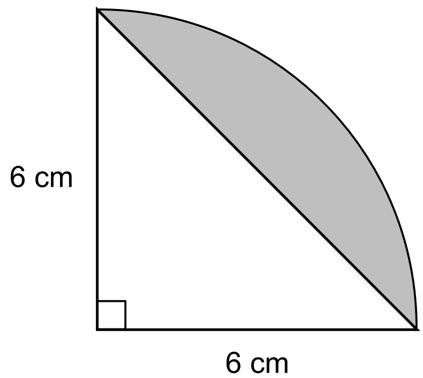
Not drawn accurately



Work out the length of the radius.

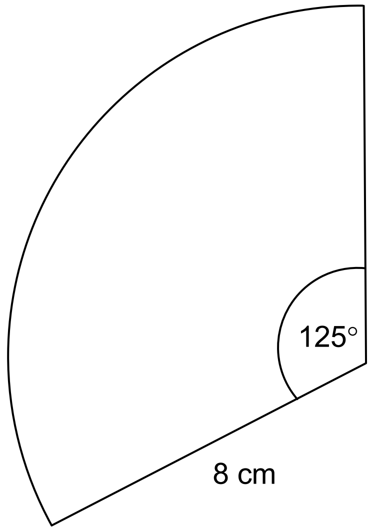
2 A quadrant of a circle is shown.

Not drawn accurately



Work out the shaded area.

3 The diagram shows a sector of a circle.



Not drawn accurately

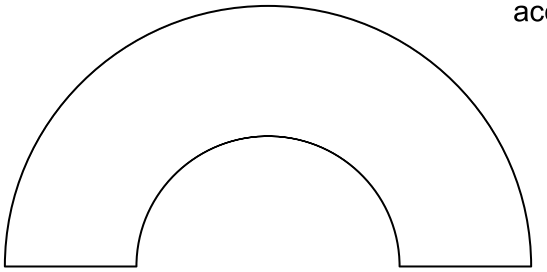
Work out the area.

4 The diagram shows a shape made from two semicircles with the same centre.

The outer radius is 10 cm

The inner radius is 6 cm

Not drawn accurately



Work out the perimeter of the shape.

G19

Know the formulae for: Pythagoras' theorem, $a^2 + b^2 = c^2$, and the trigonometric ratios, $\sin\theta$, $\cos\theta$ and $\tan\theta$; apply them to find angles and lengths in right-angled triangles in 2D figures

Teaching Guidance

Students should be able to:

- understand, recall and use Pythagoras' theorem in 2D problems
- understand, recall and use trigonometric relationships in right-angled triangles
- use the trigonometric relationships in right-angled triangles to solve problems, including those involving bearings.

Notes

Questions may be set in context, for example, a ladder against a wall.

Examples

1 Use Pythagoras' theorem to find the height of a triangle and then use the result to find the perimeter or area of the triangle.

2 Work out the amount of fencing needed to cut off a given triangular area from the corner of a field.

3 Find unknown sides or angles in a right-angled triangle.
(Diagram given)

G19e

Know the formulae for: Pythagoras' theorem, $a^2 + b^2 = c^2$, and the trigonometric ratios, $\sin\theta$, $\cos\theta$ and $\tan\theta$; apply them to find angles and lengths in right-angled triangles in 3D figures

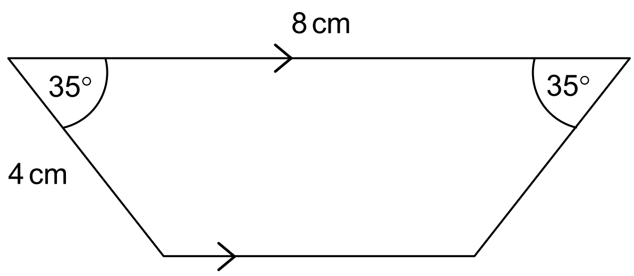
Teaching Guidance

Students should be able to:

- understand and use Pythagoras' theorem and trigonometry in complex 2D problems
- understand and use Pythagoras' theorem in 3D problems
- understand and use trigonometric relationships in 3D problems
- use these relationships in 3D contexts, including finding the angle between a line and a plane.

Examples

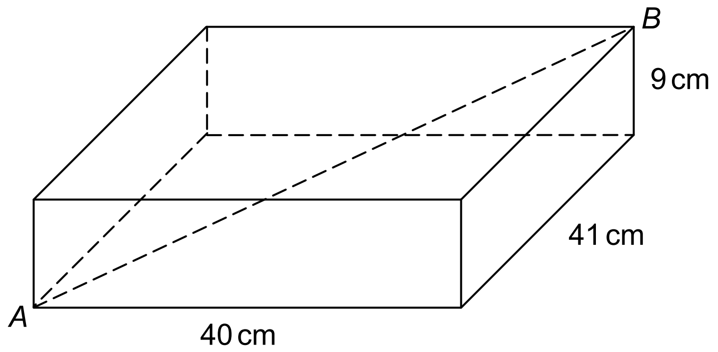
1 The diagram shows an isosceles trapezium.



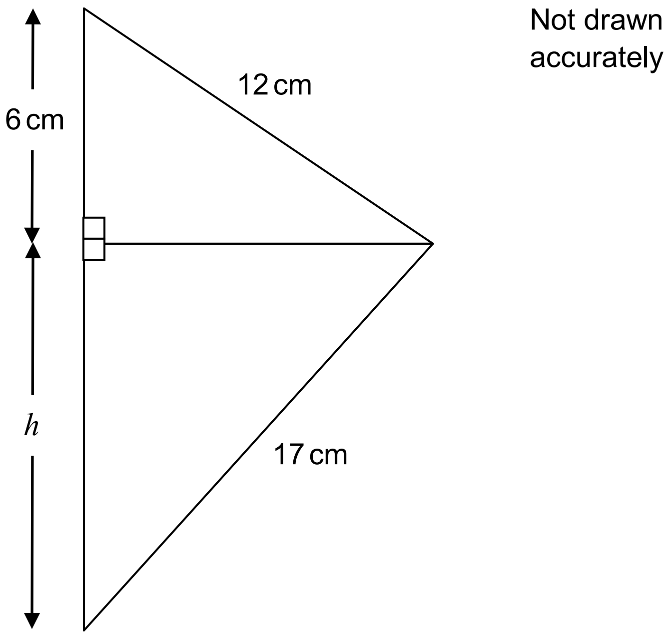
Not drawn accurately

Work out the area of the trapezium.

2 Work out the length of the diagonal *AB* in a cuboid with dimensions 9 cm, 40 cm and 41 cm



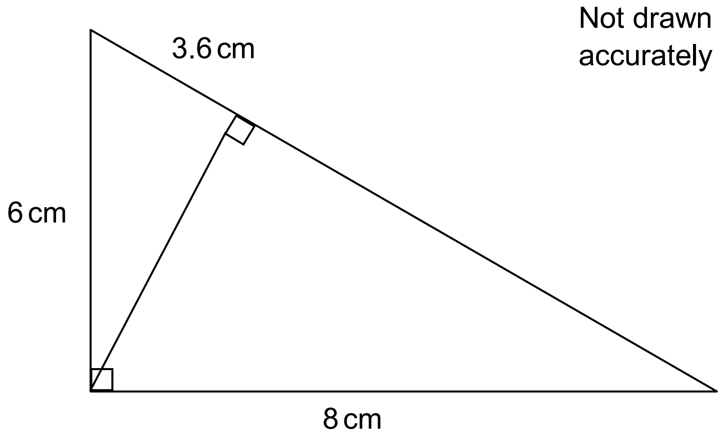
3 This diagram shows two right-angled triangles.



(a) Work out the length marked h on the diagram.

(b) Work out the area of the whole shape.

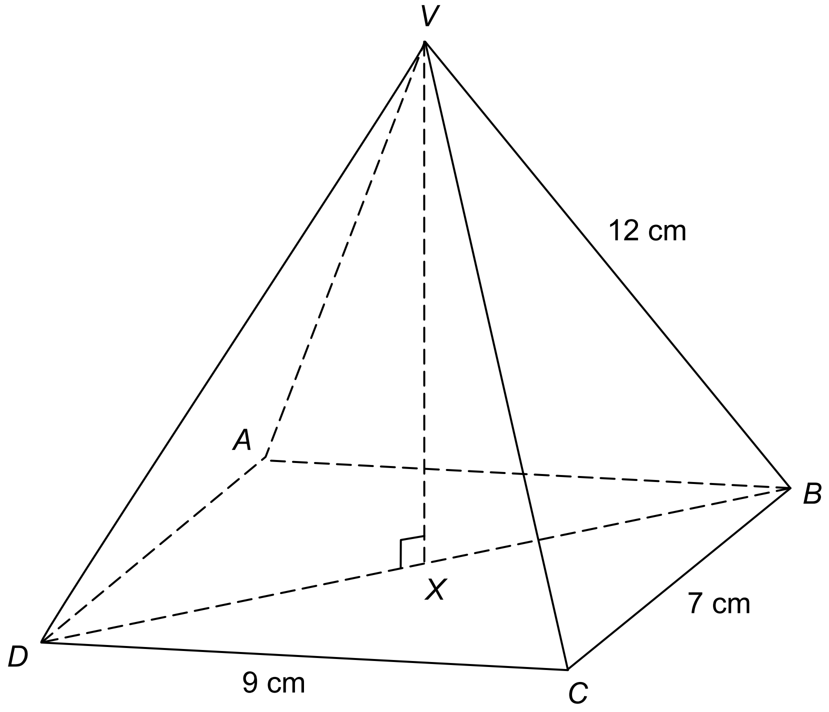
4 Two triangles are placed alongside one another, as shown in the diagram below.



Work out the area of the larger of these two triangles.

5 $VA = VB = VC = VD = 12$ cm

$ABCD$ is a rectangle 9 cm by 7 cm



(a) Work out the length VX.

(b) Work out the size of the angle between VB and the base $ABCD$.

G20e

Know and apply the sine rule, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ and cosine rule, $a^2 + b^2 = c^2 - 2bc \cos A$, to find unknown lengths and angles; know and apply Area $= \frac{1}{2}ab \sin C$ to calculate the area, sides or angles of any triangle

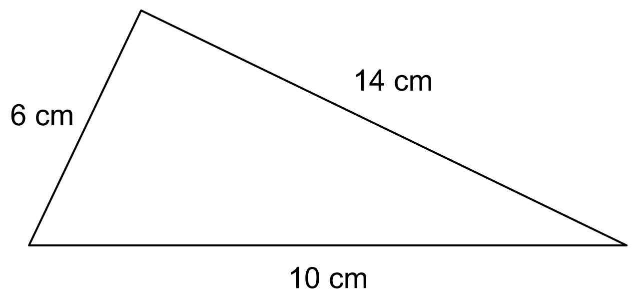
Teaching Guidance

Students should be able to:

- use the sine and cosine rules to solve 2D and 3D problems
- calculate the area of a triangle using $\frac{1}{2}ab \sin C$
- calculate the area of a triangle given the length of two sides and the included angle.

Examples

1 Find the largest angle of a scalene triangle with sides of 6 cm, 10 cm and 14 cm



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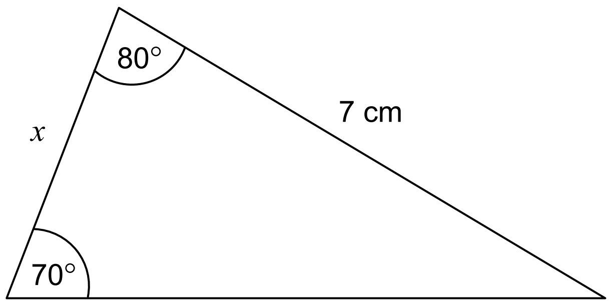
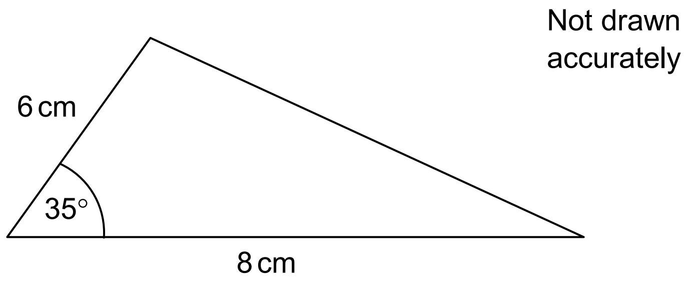
2



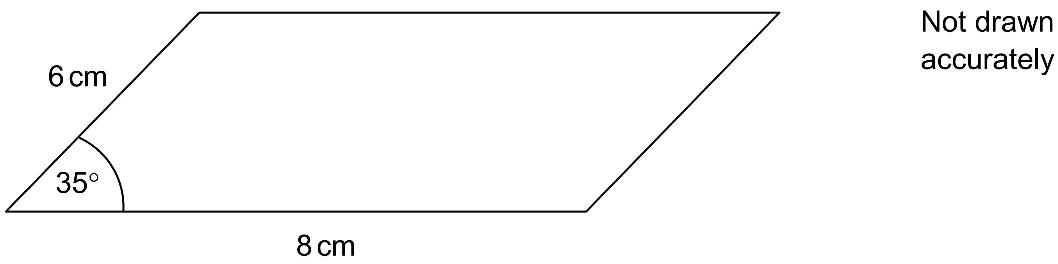
Not drawn accurately

Work out the value of x.

3 Work out the area of the triangle.



4 Work out the area of the parallelogram.



G21

Describe and transform 2D shapes using single rotations, reflections, translations or enlargements by a positive scale factor and distinguish properties that are preserved under particular transformations

Teaching Guidance

Students should be able to:

- describe and transform 2D shapes using single rotations
- understand that rotations are specified by a centre and an angle
- find a centre of rotation
- rotate a shape about the origin or any other point
- measure the angle of rotation using right angles
- measure the angle of rotation using simple fractions of a turn
- describe and transform 2D shapes using single reflections
- understand that reflections are specified by a mirror line
- find the equation of a line of reflection
- describe and transform 2D shapes using translations
- understand that translations are specified by a distance and direction (using a vector)
- translate a given shape by a vector
- describe and transform 2D shapes using enlargements by a positive scale factor
- understand that an enlargement is specified by a centre and a scale factor
- draw an enlargement
- find the centre of enlargement
- enlarge a shape on a grid (centre not specified)
- recognise that enlargements preserve angle but not length
- identify the scale factor of an enlargement of a shape as the ratio of the lengths of two corresponding sides
- identify the scale factor of an enlargement as the ratio of the lengths of any two corresponding line segments
- distinguish properties that are preserved under particular transformations
- understand that lengths and angles are preserved under rotation, reflection and translation, so that any figure is congruent under any of these transformations.

Notes

The direction of rotation will be given, unless 180°

Column vector notation should be understood.

Enlargements may be drawn on a grid, or on a Cartesian grid, where the centre of enlargement will always be at the intersection of two grid lines.

When describing transformations, the minimum requirement is:

- rotations described by centre, direction (unless half a turn) and an amount of turn (as a fraction of a whole or in degrees)
- reflections described by a mirror line
- translations described by a vector or a clear description such as three squares to the right, five squares down
- enlargements described by centre of enlargement and scale factor.

Examples

1 Enlarge a shape given on a grid by scale factor 2 and identify the centre of enlargement used.

2 Enlarge a shape given on a grid by scale factor $\frac{1}{2}$ and identify the centre of enlargement used.

3 Given a transformation from shape *A* to shape *B*, describe the reverse transformation.

4 Given two shapes (eg squares) where different transformations are possible, describe the different possible transformations.

5 Grid with diagram showing shape A given.

The vector to translate shape A to shape B is $\begin{pmatrix} 5 \\ -1 \end{pmatrix}$

Draw shape B.

6 The vector to translate shape A to shape B is $\begin{pmatrix} 5 \\ -1 \end{pmatrix}$

Write down the vector for translating shape B to shape A.

7 Draw a right-angled triangle on the grid and then translate the triangle by vector $\begin{pmatrix} 5 \\ -1 \end{pmatrix}$

Label your original triangle A and your new triangle B.

G21e

Describe and transform 2D shapes using combined transformations and enlargements by negative scale factors

Teaching Guidance

Students should be able to:

- describe a combination of transformations as a single transformation
- map a point on a shape under a combination of transformations
- identify the scale factor of an enlargement
- construct enlargements with negative scale factors.

Notes

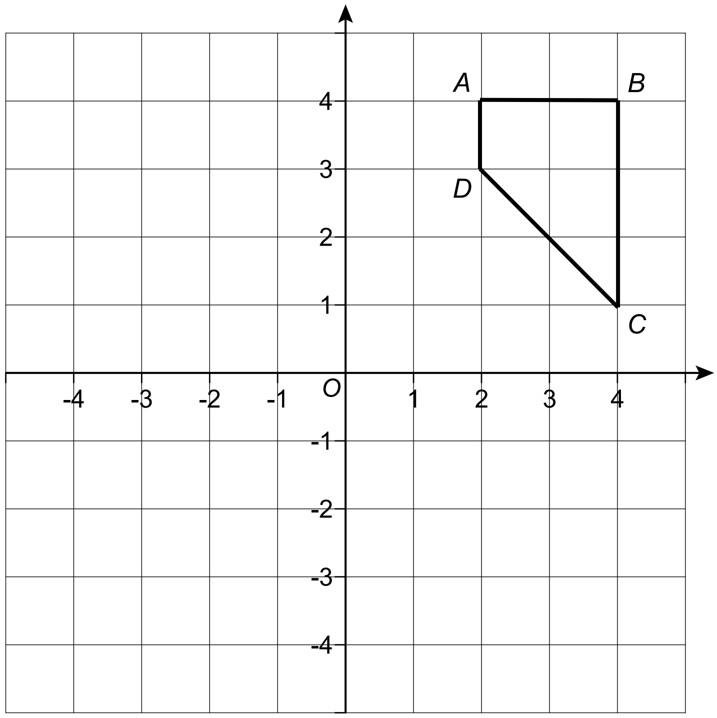
Enlargements may be drawn on a grid, or on a Cartesian grid.

Examples

1 Enlarge a shape given on a grid by scale factor –2 and identify the centre of enlargement used.

2 Enlarge a shape given on a grid by scale factor $-\frac{1}{3}$ with a given centre of enlargement.

3

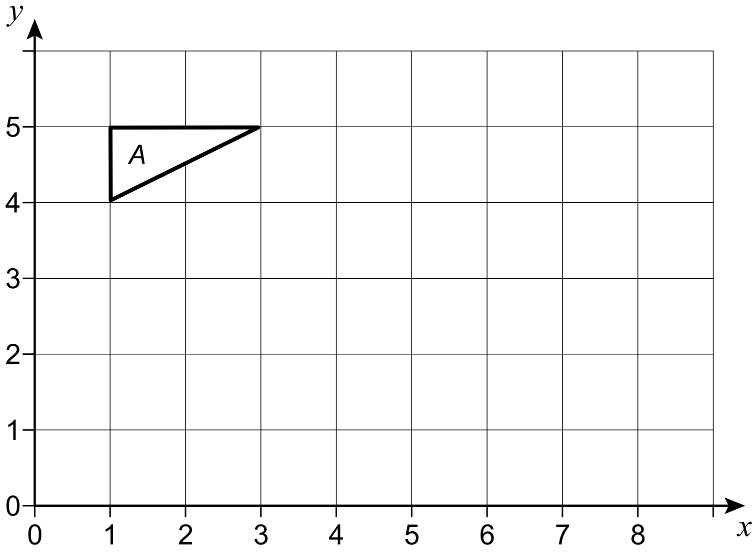


Work out the coordinates of *B* after

(a) *ABCD* is reflected in the line $y = x$, and then in the x-axis.

(b) *ABCD* is rotated through 90° anticlockwise about the point (3, 0) and then translated by the vector $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$

4



(a) Reflect *A* in the line $y = x$

Label your shape *B*.

(b) Reflect *B* in the line $x = 5$

Label your shape *C*.

(c) Describe fully the **single** transformation that maps *A* onto *C*.

G22e

Understand and use vector notation; calculate and represent graphically the sum of two vectors, the difference of two vectors and a scalar multiple of a vector; understand and use the commutative and associative properties of vector addition; solve simple geometrical problems in 2D using vector methods

Teaching Guidance

Students should be able to:

- understand and use vector notation
- calculate and represent graphically the sum of two vectors, the difference of two vectors and a scalar multiple of a vector
- calculate the resultant of two vectors
- understand and use the commutative and associative properties of vector addition
- solve simple geometrical problems in 2D using vector methods
- apply vector methods for simple geometric proofs
- recognise when lines are parallel using vectors
- recognise when three or more points are collinear using vectors
- use vectors to show three or more points are collinear.

Notes

Use of bold type and arrows such as $\mathbf{a} = \overrightarrow{OA}$ will be used to represent vectors in geometrical problems.

Examples

1 Find possible coordinates of the fourth vertex of a parallelogram with vertices at (2, 1), (–7, 3) and (5, 6) (Diagram given)

2 In a quadrilateral *ABCD* $\overrightarrow{AB} = \overrightarrow{DC}$

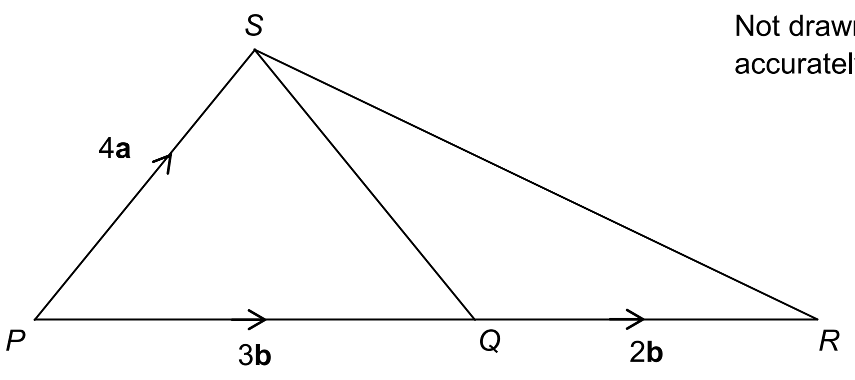
What is the special name of the quadrilateral?

3 In a quadrilateral *ABCD* $\overrightarrow{AB} = 3\overrightarrow{DC}$

What is the special name of the quadrilateral?

4

Not drawn accurately



(a) Write $\overrightarrow{SQ}$ in terms of **a** and **b**

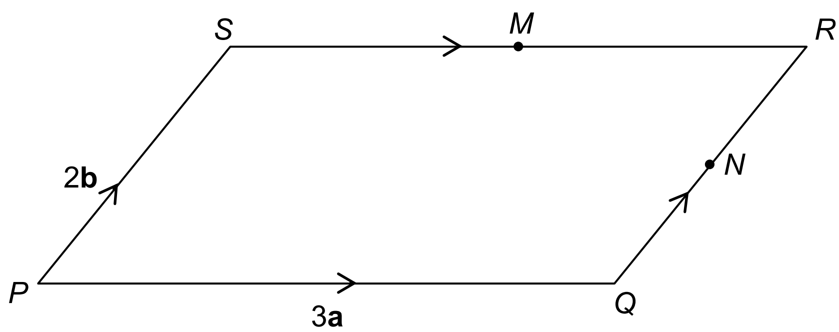
(b) Write $\overrightarrow{RS}$ in terms of **a** and **b**

5 *PQRS* is a parallelogram.

M is the midpoint of $\overrightarrow{SR}$

N is the midpoint of $\overrightarrow{QR}$

Not drawn accurately



(a) Write $\overrightarrow{SQ}$ in terms of **a** and **b**

(b) Prove that $\overrightarrow{MN}$ is parallel to $\overrightarrow{SQ}$

G23e

Multiplication of matricies

Teaching Guidance

Students should be able to

- multiply a 2 × 2 matrix by a 2 × 1 matrix
- multiply a 2 × 2 matrix by a 2 × 2 matrix
- multiply 2 × 2 and 2 × 1 matrices by a scalar
- understand that, in general, matrix multiplication is not commutative
- understand that matrix multiplication is associative.

Notes

Questions will involve finding the image (or object) point under a given transformation matrix.

Elements of matrices could be numerical or algebraic.

Questions may be set using index notation.

Candidates will be expected to be familiar with the process of equating elements of equal matrices.

Candidates will **not** be expected to find the inverse of a 2 × 2 matrix

Examples

1 $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} 1 & -2 \\ 3 & 1 \end{pmatrix}$ $\mathbf{C} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$

Work out

(a) 3**A** (b) **AB** (c) 2**BA** (d) **BC** (e) $\mathbf{A}^2$

2 Matrix $\mathbf{A} = \begin{pmatrix} 5 & 3 \\ 2 & -1 \end{pmatrix}$

Work out the image of the point (1, –2) using the transformation represented by **A**.

3 The image of a point P (x, y) is (9, 1) using the transformation represented by $\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$

Work out x and y.

4 Given $\begin{pmatrix} 2 & 4 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ a \end{pmatrix} = \begin{pmatrix} 9 + b \\ 3b \end{pmatrix}$

Work out a and b.

G24e **The identity matrix**

Teaching Guidance

Students should be able to

- understand that **AI** = **IA** = **A**

Notes

The identity matrix, **I**, will be used in the context of matrix transformations.

Candidates will **not** be expected to find the inverse of a 2 × 2 matrix.

Examples

1 $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Show that $\mathbf{A}^2 = \mathbf{I}$

2 $\mathbf{A} = \begin{pmatrix} 0 & 4 \\ 2 & -1 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} 3 & 12 \\ 6 & 0 \end{pmatrix}$

Show that $\mathbf{AB} = 24\mathbf{I}$

G25e

Transformations of the unit square in the x–y plane

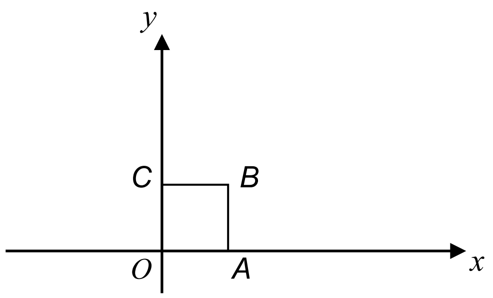
Teaching Guidance

Students should be able to

- work out the image of any vertex of the unit square given the matrix operator
- work out or recall the matrix operator for a given transformation

Notes

Transformations will be limited to transforming the unit square.



Transformations of the unit square will be restricted to rotations through 90°, 180° and 270° about the origin, reflections in the axes and the lines $y = x$ and $y = -x$, and enlargements centred on the origin.

For enlargements the term scale factor should be known. Both positive and negative scale factors will be used.

Candidates will be expected to understand the notation *A′* to mean the image of a point *A* under a transformation.

Candidates will not be expected to understand the phrase 'invariant point'.

The knowledge and use of unit vectors **i** and **j** is not required.

Examples

1 *A* (1, 0) and *C* (0, 1) are opposite vertices of the unit square *OABC*.

The square is mapped to *OA'B'C'* under transformation matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

Work out the coordinates of *A'*, *B'*, and *C'*.

2 The matrix **M** represents a reflection in the line $y = -x$.

Work out **M**.

3 The matrix **M** = 3**I**

Describe geometrically the transformation represented by **M**.

G26e

Combinations of transformations

Assessment Guidance

Students should be able to

- understand that the matrix product **PQ** represents the transformation with matrix **Q** followed by the transformation with matrix **P**
- use the skills of G23e to work out the matrix which represents a combined transformation.

Notes

Transformations will be limited to transforming the unit square.

Transformations will be restricted to those listed in G25e.

A combined transformation may be referred to as a composite transformation.

Examples

1 The matrix **M** = $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

Describe geometrically the transformation represented by

(a) **M**

(b) $\mathbf{M}^2$

2 Here are two transformations in the x-y plane:

A: Reflection in the x-axis

B: Rotation clockwise about the origin through 90º.

(a) Work out the single matrix which represents the composite transformation **A** followed by **B**.

(b) Does the composite transformation **B** followed by **A** give the same image of a shape as **A** followed by **B**?

Explain your answer.

3 The matrix **A** is such that $\mathbf{A}^2 = \mathbf{I}$ and $\mathbf{A} \neq \mathbf{I}$

By considering transformations in the x-y plane, work out a possible matrix for **A**.

4 Here are three transformations in the x-y plane:

A: Reflection in the x-axis

B: Reflection in the y-axis

C: Rotation about *O* through 180°.

Use matrix multiplication to prove that **C** is the same as **A** followed by **B**.

S1 **Understand and use qualitative, discrete and continuous data, including grouped and ungrouped data**

Teaching Guidance

Students should be able to:

- understand that samples may or may not be representative of a population
- understand that the size and construction of a sample will affect how representative it is
- understand the difference between grouped and ungrouped data
- understand the advantages and disadvantages of grouping data
- distinguish between primary and secondary data.

Examples

1 The times taken to run a race are recorded.

Is the data discrete or continuous?

2 The shoe sizes of students in a class are recorded.

Is the data discrete or continuous?

S2 **Extract data from printed tables and lists**

Teaching Guidance

Students should be able to:

- understand data given in various forms.

Examples

1 The table shows the gender of students in each year group in a school.

Year	**7**	**8**	**9**	**10**	**11**
Male	82	89	101	95	92
Female	75	87	87	99	101

(a) Which year group has the most students?

(b) What percentage of Year 9 students are boys?

(c) A student from the school is chosen at random to welcome a visitor.

What is the probability this student is a Year 7 girl?

2 The data shows the number of passengers on bus services during one morning.

29	**45**	**43**	**38**	**29**	**21**	**14**	**12**	**11**	**7**	**12**

Buses are every 30 minutes starting from 7 am.

(a) What time was the bus with the fewest passengers?

(b) Estimate the time of the morning ‘rush hour’.

(c) Give a reason for your answer to part (b).

S3	**Design and use two-way tables for grouped and ungrouped data**

Teaching Guidance

Students should be able to:

- design and use two-way tables
- complete a two-way table from given information.

Examples

1 Here is a list of vehicles that passed a checkpoint.

Red van	White car	Grey car	Grey lorry	White car	White van	White lorry
Grey car	White car	Red van	White van	White van	Grey lorry	Red car
White lorry	White car	Red car	White van	White car	White car	Grey lorry

Construct a two-way table to show the data.

2 The table shows the number of shoppers the weekend before a sale and the weekend of the sale.

	Saturday	**Sunday**
Weekend before sale	675	389
Weekend of sale	741	419

Does the data provide evidence to support a claim of a 10% increase in shoppers during the sale?

S4 | **Produce charts and diagrams for various data types; scatter graphs, stem-and-leaf, tally charts, pictograms, bar charts, dual and composite bar charts, pie charts, line graphs, frequency polygons, histograms with equal class intervals**

Teaching Guidance

Students should be able to:

- draw any of the above charts or diagrams
- understand which of the diagrams are appropriate for different types of data
- interpret any of the types of diagram
- obtain information from any of the types of diagram
- understand that a time series is a series of data points typically spaced over uniform time intervals
- plot and interpret time-series graphs
- use a time-series graph to predict a subsequent value
- understand that if data points are joined with a line then the line will not represent actual values but will show a trend
- recognise and name positive, negative or no correlation as types of correlation.

Examples

1 The table shows the gender of students in each year group in a school.

Year	**7**	**8**	**9**	**10**	**11**
Male	82	89	101	95	92
Female	75	87	87	99	101

Show this data on a suitable diagram.

(Graph paper provided with no axes or labels)

2 The table shows information about a large flock of sheep.

	Sheared sheep	**Unsheared sheep**
Black sheep	24	16
White sheep	176	264

Draw a pie chart to illustrate the data.

S4e

Produce charts and diagrams including histograms with unequal class intervals, cumulative frequency diagrams, box plots

Teaching Guidance

Students should be able to:

- understand which diagrams are appropriate for different types of data
- construct suitable diagrams for grouped discrete and continuous data.

Examples

1 The table shows the time taken for 100 runners to finish a fun run.

Time, t (minutes)	**Frequency**
$10 < t \leqslant 20$	8
$20 < t \leqslant 30$	26
$30 < t \leqslant 40$	51
$40 < t \leqslant 50$	15

Draw a cumulative frequency diagram for the data.

2 The table shows the length of 100 bolts.

Length, l (mm)	**Frequency**
$10 < l \leqslant 11$	12
$11 < l \leqslant 11.5$	25
$11.5 < l \leqslant 12$	30
$12 < l \leqslant 13$	19
$13 < l \leqslant 15$	14

Show this information in an appropriate diagram.

3 Here is some information about the marks of students in a test.

Lowest score 12, lower quartile 17, median 24, interquartile range 13, range 28

Draw a boxplot to show the information.

S5 **Calculate median, mean, range, mode and modal class**

Teaching Guidance

Students should be able to:

- find the mean for a discrete frequency distribution
- find the median for a discrete frequency distribution
- find the mode or modal class for frequency distributions
- calculate an estimate of the mean for a grouped frequency distribution, knowing why it is an estimate
- find the interval containing the median for a grouped frequency distribution.

Examples

1 Bags of crisps are labelled as containing 25 g

20 bags are sampled and their weights, measured to the nearest gram, are shown below.

26	**25**	**26**	**27**	**24**	**26**	**25**	**26**	**25**	**22**
26	**26**	**25**	**24**	**27**	**26**	**25**	**25**	**25**	**25**

(a) Work out the mode.

(b) Work out the median.

(c) Work out the mean.

(d) Comment on the label of 25 g based on the data and your results from parts (a), (b) and (c).

2 The table shows the heights of 100 five-year-old boys.

Height, h (cm)	**Frequency**
$80 \leqslant h < 90$	8
$90 \leqslant h < 100$	31
$100 \leqslant h < 110$	58
$110 \leqslant h < 120$	3

(a) Calculate an estimate of the mean height of these boys.

(b) Give a reason why your answer to part (a) is an estimate.

S5e | **Calculate quartiles and interquartile range and percentiles**

Teaching Guidance

Students should be able to:

- calculate quartiles and interquartile range from a small data set using the positions of the lower quartile and upper quartile respectively (the number of entries in a small data set will be 1 less than a multiple of 4) $\frac{n+1}{4}$ and $\frac{3(n+1)}{4}$ should be used for the upper and lower quartiles respectively
- other methods such as Tukey's hinge should not be used
- read off lower quartile, median and upper quartile from a cumulative frequency diagram or a box plot and calculate the interquartile range
- find an estimate of the median or other information from a histogram
- choose an appropriate measure according to the nature of the data to be the 'average'.

Examples

1 From a cumulative frequency diagram:

Estimate the value of the inter-quartile range.

2 (Cumulative frequency diagram given and some information about the max and min.)

Use the diagram to produce a further diagram that will show the spread of the distribution.

3 Work out the interquartile range of this set of data.

12 14 9 9 15 14 17 10 11 12 15

12 Statistics and probability – Interpretation

S6	**Read and interpret a wide range of graphs and diagrams and draw conclusions**

Teaching Guidance

Students should be able to:

- interpret diagrams for grouped discrete and continuous data.
- draw a conclusion about a distribution from a diagram

Examples

1 Given a pie chart about votes in a talent competition:

(a) Name the winner

(b) Work out their share of the vote.

2 Given a bar chart for daily sales of handbags by a shop over one week, and profit per handbag, work out the total profit.

Note

The above examples apply to both Core and extension.

Reading and interpreting the following will only be tested on extension.

- histograms with unequal class intervals
- cumulative frequency diagrams
- boxplots

Example

1 Histogram given for ages of people.

Use the histogram to work out the number of people under 60 years old.

S7 **Compare distributions and make inferences**

Teaching Guidance

Students should be able to:

- compare two distributions in order to make inferences
- compare two diagrams in order to make decisions about a hypothesis.

Examples

1 The table shows the gender of students in each year group in a school.

Year	7	8	9	10	11
Male	82	89	101	95	92
Female	75	87	87	99	101

Compare the data for the boys with the data for girls.

Note

The above examples apply to both Core and extension.

Comparing distributions and making inferences for the following will only be tested on extension.

- histograms with unequal class intervals
- cumulative frequency diagrams
- boxplots

Example

1 From two box plots:

Compare the data for the yield of plants with and without fertiliser.

(Median and inter-quartile range comparisons expected)

S8 | **Recognise correlation and draw and/or use lines of best fit by eye, understanding what these represent**

Teaching Guidance

Students should be able to:

- recognise and name positive, negative or no correlation as types of correlation
- recognise and name strong, moderate or weak correlation as strengths of correlation
- understand that just because a correlation exists, it does not necessarily mean that causality is present
- draw a line of best fit by eye for data with strong enough correlation, or know that a line of best fit is not justified due to the lack of correlation
- understand outliers and make decisions whether or not to include them when drawing a line of best fit
- use a line of best fit to estimate unknown values when appropriate.
- make predictions, interpolate and comment on the dangers of extrapolation.

Examples

1 From a scatter diagram:

(a) Write the down the strength **and** type of correlation shown by the diagram.

(b) Interpret your answer to part (a) in the context of the question.

S9 **Understand and use the vocabulary of probability and the probability scale**

Teaching Guidance

Students should be able to:

- use words to indicate the chances of an outcome for an event
- use fractions, decimals or percentages to put values to probabilities
- place probabilities or outcomes to events on a probability scale.

Notes

The words candidates should be familiar with will be limited to impossible, (very) unlikely, evens or even chance, (very) likely and certain.

Candidates should **not** use word forms or ratio such as 1 out of 2 or 1 : 2 for numerical probabilities

Examples

1 Circle the appropriate probability word for each event.

(a) The chance of a goat passing GCSE Mathematics

Impossible Unlikely Even chance Likely Certain

(b) The chance it will rain next week at your house

Impossible Unlikely Even chance Likely Certain

2 Which of these values could not represent a probability?

0.6 1.2 –0.05 $\frac{4}{3}$ $\frac{4}{5}$

3 Look at these events for an ordinary fair dice.

A roll the number 1

B roll a 7

C roll a number less than 7

Draw a probability scale.

Indicate the positions of the probabilities for events A, B and C.

S10 **Understand and use estimates or measures of probability from theoretical models (including equally likely outcomes) or from relative frequency**

Teaching Guidance

Students should be able to:

- understand and use the term relative frequency
- consider differences where they exist between the theoretical probability of an outcome and its relative frequency in a practical situation
- recall that an ordinary fair dice is an unbiased dice numbered 1, 2, 3, 4, 5 and 6 with equally likely outcomes
- estimate probabilities by considering relative frequency.

Examples

1 A bag contains blue counters, red counters and green counters.
The probability of picking a blue counter = the probability of picking a red counter
The probability of picking a green counter = 0.3

Complete this table.

Colour	Number of counters
Blue	14
Red	
Green	

2 United have won 12 of the last 20 games they played.

(a) What is the relative frequency of wins?

(b) Use your answer to part (a) to estimate the probability that United will win their next game.

(c) Why may this not be a good method to use for estimating this probability?

S11 **Compare experimental data and theoretical probabilities**

Teaching Guidance

Students should be able to:

- understand and use the term relative frequency
- consider differences where they exist between the theoretical probability of an outcome and its relative frequency in a practical situation.

Notes

To be considered in conjunction with the issues from S12 and 13

Examples

1 A fair dice is rolled 60 times.

(a) How many times would you expect to see a 6 rolled?

(b) Why is it unlikely that you would see your answer to part (a) occurring?

2 In an experiment, a rat turns either left or right in a maze to find food.
After 200 experiments, the relative frequency of the rat turning left was 0.45

How many times did the rat turn right in the 200 experiments?

S12 **Understand that if an experiment is repeated, this may – and usually will – result in different outcomes**

Teaching Guidance

Students should be able to:

- understand that experiments rarely give the same results when there is a random process involved
- appreciate the 'lack of memory' in a random situation, for example a fair coin is still equally likely to give heads or tails even after five heads in a row.

Notes

To be considered in conjunction with the issues from S11 and S13

Examples

1 A fair dice is rolled several times.

Here are some of the results.

4 6 2 4 3 1 1 1 1 1

On the next roll, what is the probability of a 1?

S13

Understand that increasing sample size generally leads to better estimates of probability and population characteristics

Teaching Guidance

Students should be able to:

- understand that the greater the number of trials in an experiment the more reliable the results are likely to be
- understand how a relative frequency diagram may show a settling down as sample size increases, enabling an estimate of a probability to be reliably made; and that if an estimate of a probability is required, the relative frequency of the largest number of trials available should be used.

Notes

Refer also to S11 and S12

Examples

1 From a relative frequency diagram:

Use the diagram to make the best estimate of the probability of picking a red disc.

2 Aisha catches 10 frogs at random from a pond and measures their weight.

She then uses the data to estimate the mean weight of a frog in the pond.

How could she obtain a more reliable estimate for this mean?

3 The table shows the number of heads obtained in every 10 flips of a coin.

Trials	**1st 10**	**2nd 10**	**3rd 10**	**4th 10**	**5th 10**
Number of heads	3	2	2	1	2

Draw a relative frequency graph for this data (graph paper available).

Use your graph or otherwise to obtain an estimate of the probability of a head for this coin.

S14

Understand and use sample spaces for situations where outcomes are single events and for situations where outcomes are two successive events

Teaching Guidance

Students should be able to:

- list all the outcomes for a single event in a systematic way
- list all the outcomes for two events in a systematic way
- design and use two-way tables
- complete a two-way table from given information
- design and use sample spaces
- work out probabilities by counting or listing equally likely outcomes.

Examples

1 A fair dice is rolled twice.

Show all the possible total scores in a two-way table.

(Outline usually given)

Use the table to find the probability that the total is 10

2 A drinks machine sells Tea (T), Coffee (C) and Soup (S).

Gareth buys 2 drinks at random.

List all the possible pairs of drinks he could buy.

Use your list to find the probability that both drinks are the same.

3 The coins of a country are 1 cent, 5 cents, 10 cents, 25 cents and 50 cents.

Jane has two of the same type of coin.

Work out the probability that she has at least 40 cents in total.

S15

Identify different mutually exclusive and exhaustive outcomes and know that the probabilities of all of these outcomes is 1

Know and use that for mutually exclusive events A and B, P(AUB) = P(A) + P(B)

Teaching Guidance

Students should be able to:

- determine when it is appropriate to add probabilities
- understand when outcomes can or cannot happen at the same time
- use this understanding to calculate probabilities
- appreciate that the sum of the probabilities of all possible mutually exclusive outcomes has to be 1
- find the probability of a single outcome from knowing the probability of all other outcomes.

Examples

1 Dan wants a ‘Ben 10’ watch for this birthday.

The probability that his gran will buy him one is 0.4

The probability that his brother will buy him one is 0.6

(a) What is the probability that both his brother and his gran buy Dan the watch?

(b) What is the assumption you made to answer part (a)?

(c) Is this a fair assumption to make in this context?

2 A spinner can land on either 1, 2, 3 or 4.
Some of the probabilities are shown in the table.

Value	Probability
1	0.274
2	
3	0.216
4	0.307

Work out the missing probability.

3 Sort these dice outcomes into pairs that **can** happen at the same time.

A rolling a 6

B rolling an odd number

C rolling a number more than 5

D rolling a 4

E rolling an even square number

F rolling a 1

4 The probability that Andy passes his driving test is 0.67

Work out the probability that Andy does **not** pass his driving test.

S16 **Understand and use Venn diagrams to work out probabilities**

Teaching Guidance

Students should be able to:

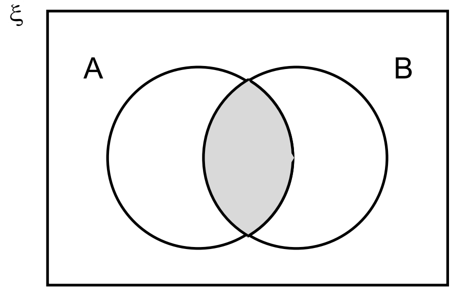
- understand that P(A) means the probability of event A
- understand that P(A$^{/}$) means the probability of event **not** A
- understand that P(A $\cup$ B) means the probability of event A or B or both
- understand that P(A $\cap$ B) means the probability of event A and B
- understand a Venn diagram consisting of a universal set and at most two sets, which may or may not intersect
- shade areas on a Venn diagram involving at most two sets, which may or may not intersect
- solve problems given a Venn diagram
- solve problems, where a Venn diagram approach is a suitable strategy to use but a diagram is not given in the question.

Note

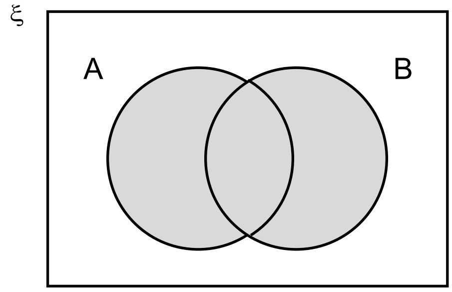
Conditional probabilities will only be tested in Extension

Notes

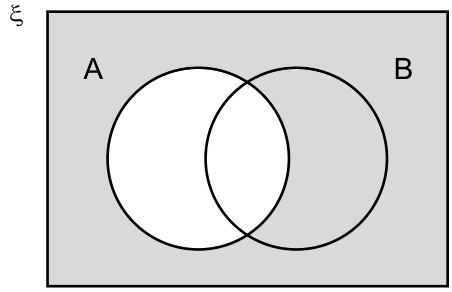
Students should know the following notations and the associated shaded area on a Venn diagram.



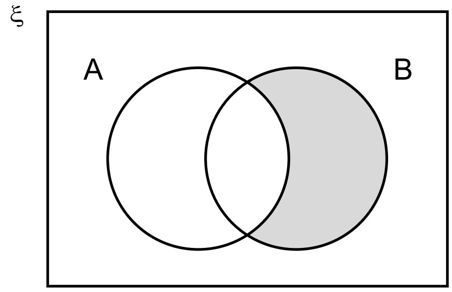
$A \cap B$ to mean the intersection of A and B



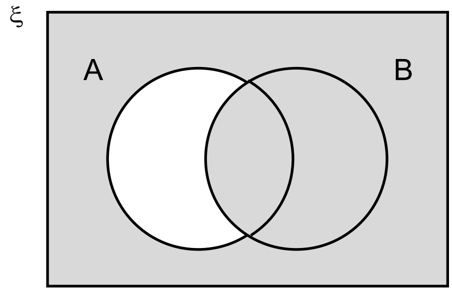
$A \cup B$ to mean the union of A and B



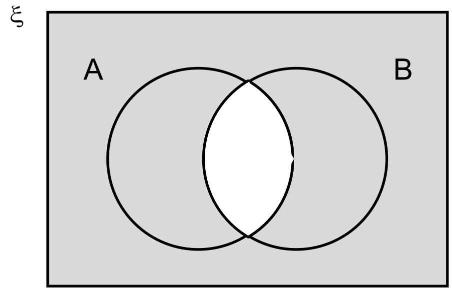
A' to mean everything not in A



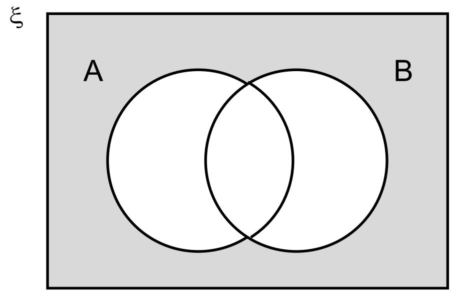
$A' \cap B$ to mean everything not in A that is in B



$A' \cup B$ to mean the union of A' and B



$(A \cap B)' = A' \cup B'$ to mean everything not in the intersection



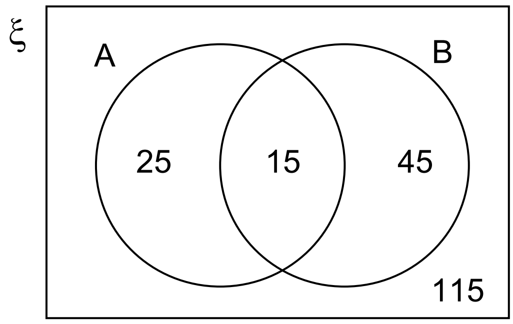
$(A \cup B)' = A' \cap B'$ to mean everything not in the union

Examples

1 You are given that $P(A) = 0.7$

Work out $P(A')$

2 The Venn diagram shows the number of left-handed students in a year group (set A) and the number of vegetarians in the same year group (set B).



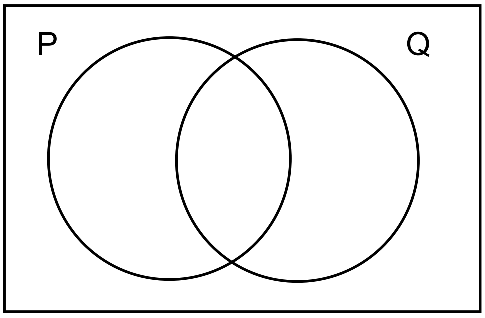
(a) Write down $P(A \cap B)$

(b) How many students are in the year group altogether?

(c) A student from the year group is chosen at random.

What is the probability that the student is a right-handed vegetarian?

3 On the Venn diagram, shade the area that represents $P' \cap Q$



4 A garage has 50 cars for sale.

16 of the cars have air conditioning and ABS brakes.

32 of the cars have air conditioning.

18 of the cars have ABS brakes.

Work out how many of the cars do **not** have air conditioning or ABS brakes.

5 A running club has 120 members.

89 of the members take part in road races.

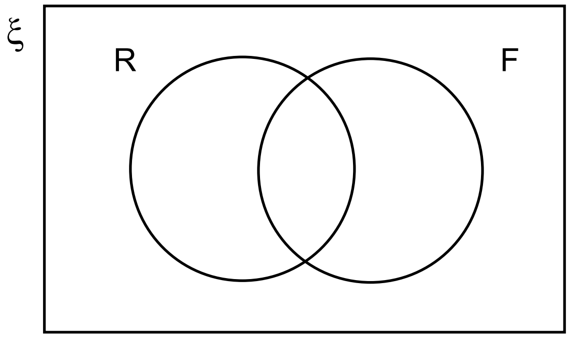
54 of the members take part in fell races.

17 of the members do not run in road or fell races.

Use this information to complete the Venn diagram.

R represents those runners who run in Road races.

F represents those runners who run in Fell races.



S17e

Calculate the probability of independent combined events, including using tree diagrams and other representations.

Know and use that for independent events, A and B

$P(A \cap B) = P(A) \times P(B)$

Teaching Guidance

Students should be able to:

- determine when it is appropriate to multiply probabilities
- understand the meaning of independence for events.

Examples

1 A car showroom has 20 cars for sale.
Eight of these cars are silver.

Calculate the probability that the next two cars sold are not silver.

2 The table shows the time taken for 100 runners to finish a fun run.

Time, t (minutes)	**Frequency**
$10 < t \leq 20$	8
$20 < t \leq 30$	26
$30 < t \leq 40$	51
$40 < t \leq 50$	15

Work out the probability that the two runners who raised the most money for charity both finished in less than 20 minutes.

S18e	**Calculate conditional probabilities including using tree diagrams and other representations**

Teaching Guidance

Students should be able to:

- understand conditional probability
- understand the implications of with or without replacement problems for the probabilities obtained
- complete a tree diagram to show outcomes and probabilities
- use a tree diagram as a method for calculating conditional probabilities
- use a Venn diagram as a method for calculating conditional probabilities.

Examples

1 A fair coin is thrown three times.

Work out the probability of throwing exactly two heads.

2 Amy and Becky want to be chosen for the hockey and netball teams.

Their estimates for the probabilities of being chosen (assumed independent) are shown in the table below.

	Hockey	Netball
Amy	0.8	0.7
Becky	0.6	0.25

Calculate the probability that

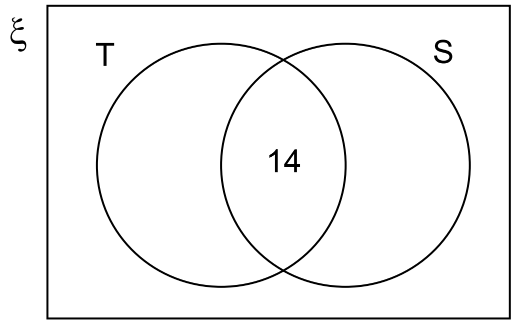
(a) Becky will be chosen for both teams

(b) Both girls are chosen for **only** the netball team.

2 80 people in a sports club were surveyed.

- 14 played tennis and squash
- 56 played tennis
- 30 played squash.

(a) Complete the Venn diagram.



(b) One person is chosen at random.

Work out the probability that

(i) the person chosen did not play tennis

(ii) the person chosen played tennis or squash or both.

(c) What is the probability that a person chosen at random who plays squash also plays tennis?

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