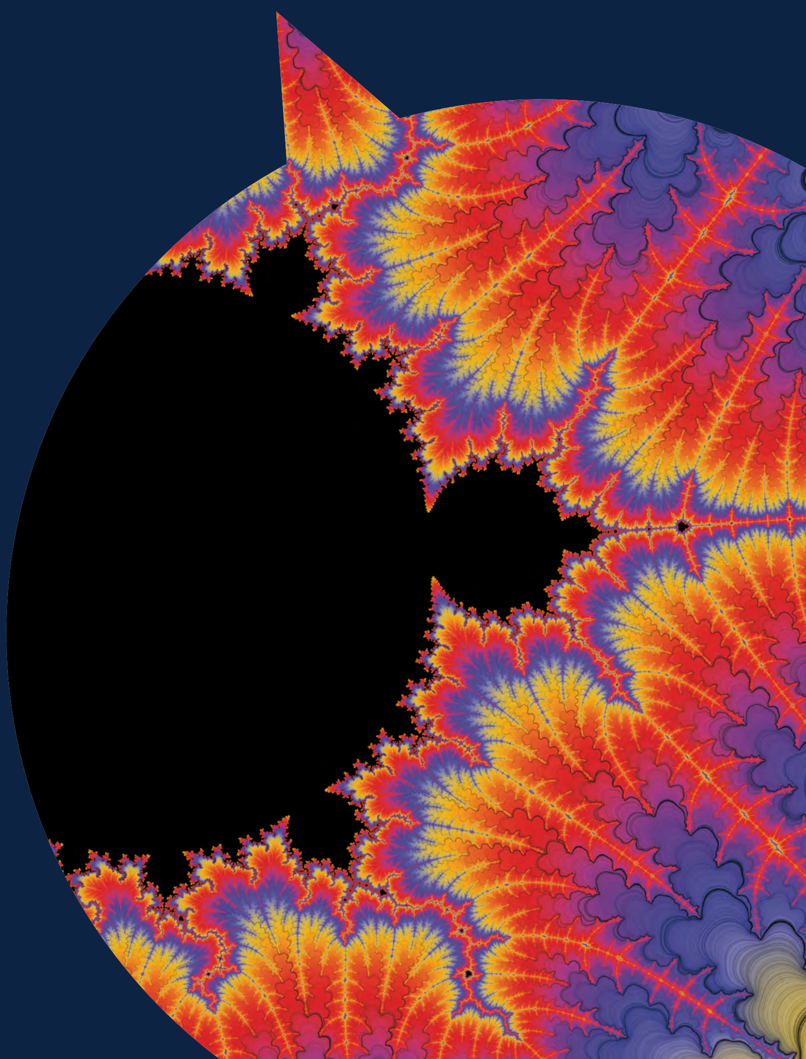
OXFORD AQA
INTERNATIONAL QUALIFICATIONS

International AS and A-level

Further Mathematics

(9665) Specification



For teaching from September 2017 onwards

For International AS exams
May/June 2018 onwards

For International A-level exams
May/June 2019 onwards

For teaching and examination outside
the United Kingdom

Contents

Are you using the latest version of this specification?

- You will always find the most up-to-date version of this specification on our website at **oxfordaqa.com/9665**
- We will write to you if there are significant changes to the specification.

1 Introduction

1.1 Why choose OxfordAQA International AS and A-levels?

Our International qualifications enable schools that follow a British curriculum to benefit from the best education expertise in the United Kingdom (UK).

Our International AS and A-levels offer the same rigour and high quality as AS and A-levels in the UK and are relevant and appealing to students worldwide. They reflect a deep understanding of the needs of teachers and schools around the globe and are brought to you by Oxford University Press and AQA, the UK's leading awarding body.

Providing valid and reliable assessments, these qualifications are based on over 100 years of experience, academic research and international best practice. They have been independently validated as being to the same standard as the qualifications accredited by the UK examinations regulator, Ofqual. They reflect the latest changes to the British system, enabling students to progress to higher education with up-to-date qualifications.

You can find out about OxfordAQA at **oxfordaqa.com**

1.2 Why choose our International AS and A-level Further Mathematics?

We have worked closely with teachers to design our specification to inspire, challenge and motivate every student, no matter what their level of ability, while supporting you in developing creative and engaging lessons.

Further maths is for those who wish to pursue the subject more deeply. It is diverse and engaging, equipping students with the right skills to reach their future destination, whatever that may be. At OxfordAQA, we design qualifications and support to enable students to engage with, explore, enjoy and succeed in maths. By putting students at the heart of everything we do, our aim is to support teachers to shape what success in maths looks like for every student.

Our question papers are designed with students in mind. We're committed to ensuring that students are settled early in our exams and have the best possible opportunity to demonstrate their knowledge and understanding of maths, to ensure they achieve the results they deserve.

The specification takes an approach to the study of mathematics that is consistent across the topic areas. Our experienced team has produced question papers and mark schemes that allow you to get back to inspirational mathematics teaching and allow students of all abilities to achieve their best on every question.

You can find out about all our International AS and A-level Further Mathematics qualifications at **oxfordaqa.com/maths**

1.3 Recognition

OxfordAQA meet the needs of international
students. Please refer to the published timetables on the exams administration page of our website **(oxfordaqa.com/exams-administration)** for up to date exam timetabling information. They are an international alternative and comparable in standard to the Ofqual regulated qualifications offered in the UK.

Our qualifications have been independently benchmarked by UK NARIC, the UK national agency for providing expert opinion on qualifications worldwide. They have confirmed they can be considered 'comparable to the overall GCE A-level and GCSE standard offered in the UK'. Read their report at **oxfordaqa.com/recognition**

To see the latest list of universities who have stated they accept these international qualifications, visit **oxfordaqa.com/recognition**

1.4 Support and resources to help you teach

We know that support and resources are vital for your teaching and that you have limited time to find or develop good quality materials. That's why we've worked with experienced teachers to provide you with resources that will help you confidently plan, teach and prepare for exams.

Teaching resources

You will have access to:

- sample schemes of work to help you plan your course with confidence
- teacher guidance notes to give you the essential information you need to deliver the specification
- training courses to help you deliver our qualifications
- student textbooks that have been checked and approved by us
- engaging worksheets and activities developed by teachers, for teachers.

Preparing for exams

You will have access to the support you need to prepare for our exams, including:

- specimen papers and mark schemes
- exemplar student answers with examiner commentaries
- a searchable bank of past AQA exam questions mapped to these new International qualifications.

Analyse your students' results with Enhanced Results Analysis (ERA)

After the first examination series, you can use this tool to see which questions were the most challenging, how the results compare to previous years and where your students need to improve. ERA, our free online results analysis tool, will help you see where to focus your teaching.

Information about results, including maintaining standards over time, grade boundaries and our post-results services, will be available on our website in preparation for the first examination series.

Help and support

Visit our website for information, guidance, support and resources at **oxfordaqa.com/9665**

You can contact the subject team directly at **maths@oxfordaqa.com** or call us on +44 (0)161 696 5995 (option 1 and then 1 again).

Please note: We aim to respond to all email enquiries within two working days.

Our UK office hours are Monday to Friday, 8am – 5pm.

2 Specification at a glance

The titles of the qualifications are:

- OxfordAQA International Advanced Subsidiary Further Mathematics
- OxfordAQA International Advanced Level Further Mathematics.

These qualifications are modular. The full International A-level is intended to be taken over two years. The specification content for the International AS is half that of an International A-level. The International AS can be taken as a stand-alone qualification or can be used to count towards the International A-level. Students can take the International AS in the first year and then take the International A2 in the second year to complete the International A-level or they can take all the units together in the same examination series at the end of the course.

The International AS content will be 50% of the International A-level content but International AS assessments will contribute 40% of the total marks for the full International A-level qualification with the remaining 60% coming from the International A2 assessments.

Candidates may re-sit a unit any number of times within the shelf-life of the specification. The best result for each unit will count towards the final qualification. Exams will be available in January and May/June.

The guided learning hours (GLH) for an OxfordAQA International Advanced Subsidiary is 180.

The guided learning hours (GLH) for an OxfordAQA International Advanced Level is 360.

These figures are for guidance only and may vary according to local practice and the learner's prior experience of the subject.

2.1 Subject content

- Pure maths
- Statistics
- Mechanics

2.2 International AS

Unit FP1 calculator allowed		Unit FPSM1 calculator allowed
What's assessed Pure maths from the FP1 content area of the specification.	+	**What's assessed** Content from the FPSM1 content area of the specification.
How it's assessed Written exam: 1 hour 30 minutes 80 marks Calculator allowed 50% of the International AS assessment 20% of the International A-level assessment		**How it's assessed** Written exam: 1 hour 30 minutes 80 marks consisting of: 40 marks Pure maths 20 marks Statistics 20 marks Mechanics Calculator allowed 50% of the International AS assessment 20% of the International A-level assessment

2.3 International A-level

International AS papers plus:

Unit FP2 calculator allowed		Unit FS2 calculator allowed		Unit FM2 calculator allowed
What's assessed Content from the FP2 area of the specification.	+	**What's assessed** Content from the FS2 area of the specification.	or	**What's assessed** Content from the FM2 area of the specification.
How it's assessed Written exam: 2 hours 30 minutes 120 marks Calculator allowed 37.5% of the International A-level assessment		**How it's assessed** Written exam: 1 hour 30 minutes 80 marks Calculator allowed 22.5% of the International A-level assessment		**How it's assessed** Written exam: 1 hour 30 minutes 80 marks Calculator allowed 22.5% of the International A-level assessment

3 Subject content

3.1 International AS Unit FP1 (Pure maths)

Students will be expected to be familiar with the knowledge, skills and understanding implicit in the unit P1 and the Pure section of PSM1 from the International AS Mathematics (9660).

Students may use relevant formulae included in the formulae booklet without proof.

FP1.1: Algebra and graphs

Content	Additional information
Graphs of rational functions of the form $\frac{ax+b}{cx+d}$, $\frac{ax+b}{cx^2+dx+e}$ or $\frac{x^2+ax+b}{x^2+cx+d}$	Sketching the graphs. Finding the equations of the asymptotes which will always be parallel to the coordinate axes. Finding points of intersection with the coordinate axes or other straight lines. Solving associated inequalities. Using quadratic theory (not calculus) to find the possible values of the function and the coordinates of the maximum or minimum points on the graph. eg for $y=\frac{x^2+2}{x^2-4x}$, $y=k \Rightarrow x^2+2=kx^2-4kx$, which has real roots if $16k^2+8k-8 \geqslant 0$, ie if $k \leqslant -1$ or $k \geqslant \frac{1}{2}$; stationary points are (1, –1) and (–2, $\frac{1}{2}$)
Graphs of parabolas, ellipses and hyperbolas with equations $y^2=4ax$, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $xy=c^2$	Sketching the graphs. Finding points of intersection with the coordinate axes or other straight lines. Students will be expected to interpret the geometrical implication of equal roots, distinct real roots or no real roots. Knowledge of the effects on these equations of single transformations of these graphs involving translations, stretches parallel to the x-axis or y-axis, and reflections in the line $y=x$ Including the use of the equations of the asymptotes of the hyperbolas given in the formulae booklet.

FP1.2: Coordinate geometry

Content	Additional information
Simple locus involving distances from points and distances from straight lines of form $x = a$ and $y = b$	eg Find the Cartesian equation of the locus of the points that are equidistant from the point (2, 3) and the line $x = 4$

FP1.3: Complex numbers

Content	Additional information
Non-real roots of quadratic equations.	Complex conjugates – awareness that non-real roots of quadratic equations with real coefficients occur in conjugate pairs.
Sum, difference and product of complex numbers in the form $x+\mathrm{i}y$	
Comparing real and imaginary parts.	Including solving equations eg $2z+z^* = 1+\mathrm{i}$ where z^* is the conjugate of z
The Cartesian and polar co-ordinate forms of a complex number, its modulus, argument and conjugate.	$x+\mathrm{i}y$ and $r(\cos\theta+\mathrm{i}\sin\theta)$
The quotient of two complex numbers.	
The representation of a complex number by a point on an Argand diagram; geometrical illustrations.	
Simple loci in the complex plane.	For example, $\lvert z-2-\mathrm{i}\rvert = 5$, $\arg(z-2)=\frac{\pi}{3}$ including $\lvert z-a\rvert = \lvert z-b\rvert$ where a and b are complex numbers.

FP1.4: Roots and coefficients of a quadratic equation

Content	Additional information
Manipulating expressions involving $\alpha+\beta$ and $\alpha\beta$	eg $\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$ Forming an equation with roots such as α^3,β^3 or $\frac{1}{\alpha},\frac{1}{\beta}$, $\alpha+\frac{2}{\beta}$, $\beta+\frac{2}{\alpha}$ etc.

FP1.5: Series

Content	Additional information
Use of formulae for the sum of the squares and the sum of the cubes of the natural numbers.	eg to find a polynomial expression for $\sum_{r=1}^{n} r^2(r+2)$ or $\sum_{r=1}^{n}\left(r^2-r+1\right)$
Summation of a finite series by method of differences.	eg $\sum_{r=1}^{n} r.r! = \sum_{r=1}^{n}\left[(r+1)!-r!\right]$
Extension to infinite series when the limit of the partial sum exists.	

FP1.6: Trigonometry

Content	Additional information
General solutions of trigonometric equations including use of exact values for the sine, cosine and tangent of $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$	eg $\sin 2x = \frac{\sqrt{3}}{2}$, $\cos\left(x+\frac{\pi}{6}\right) = -\frac{1}{\sqrt{2}}$, $\tan\left(\frac{\pi}{3}-2x\right)=1$, $\sin 2x = 0.3$, $\cos(3x-1) = -0.2$

FP1.7: Calculus

Content	Additional information
Finding the gradient of the tangent to a curve at a point, by taking the limit as h tends to zero of the gradient of a chord joining two points whose x-coordinates differ by h	The equation will be given as $y=\mathrm{f}(x)$, where $\mathrm{f}(x)$ is a simple polynomial such as x^2-2x or x^4+3
Connected rates of change and small changes.	eg $\frac{\mathrm{d}p}{\mathrm{d}t} = \frac{\mathrm{d}p}{\mathrm{d}v} \times \frac{\mathrm{d}v}{\mathrm{d}t}$ where $p = kv^{\frac{4}{3}}$ $\delta h \approx \frac{\mathrm{d}h}{\mathrm{d}x} \times \delta x$ where $h = 20x^{-2}$
Evaluation of simple improper integrals.	eg $\int_0^4 \frac{1}{\sqrt{x}}\,\mathrm{d}x$, $\int_4^{\infty} x^{-\frac{3}{2}}\,\mathrm{d}x$

3.2 International AS Unit FPSM1 (Pure maths, statistics and mechanics)

Students will be expected to be familiar with the knowledge, skills and understanding implicit in the International AS Mathematics (9660), and the unit FP1 from this specification.

Unit FPSM1 is comprised of the Pure maths FPP1, Statistics FS1 and Mechanics FM1 on the following pages.

Students may use relevant formulae included in the formulae booklet without proof.

3.2.1: Pure maths

FPP1.1: Matrices and transformations

Content	Additional information
Matrix algebra of up to 3×3 matrices, including the inverse of a 2×2 matrix.	Including non-square matrices and use of the results $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$ and $(\mathbf{AB})^{\mathrm{T}} = \mathbf{B}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}$ Singular and non-singular matrices.
The identity matrix **I** for 2 x 2 and 3 x 3 matrices.	
Transformations of points in the $x - y$ plane represented by 2×2 matrices.	Transformations will include rotations about the origin, reflections in a line through the origin, stretches parallel to the x-axis and y-axis, and enlargements with centre the origin. Use of the standard transformation matrices given in the formulae booklet. Combinations of these transformations eg $\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$, $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$, $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$, $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$
Determinant of a 2×2 matrix.	Area scale factor of transformation; interpretation of negative determinant for corresponding transformation.
Invariant points and invariant lines in 2D.	The difference between lines of invariant points and invariant lines.
Shears.	Students will be expected to recognise the matrix for a shear parallel to the x or y-axis. Where the line of invariant points is not the x or y-axis students will be informed that the matrix represents a shear.

FPP1.2: Linear graphs

Content	Additional information
Reducing a relation to a linear law.	eg $\frac{1}{x}+\frac{1}{y}=k$; $y^2=ax^3+b$; $y=ax^n$; $y=ab^x$ Use of logarithms to base 10 where appropriate. Given numerical values of (x, y), drawing a linear graph and using it to estimate the values of the unknown constants.

FPP1.3: Numerical methods

Content	Additional information
Location of roots of $\mathrm{f}(x)=0$ by considering changes of sign of $\mathrm{f}(x)$ in an interval of x in which $\mathrm{f}(x)$ is continuous.	
Finding roots of equations by interval bisection, linear interpolation and the Newton-Raphson method.	Graphical illustration of these methods.
Solving differential equations of the form $\frac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{f}(x, y)$	Using Euler's step-by-step method based on the linear approximation $y_{n+1} \approx y_n + h\,\mathrm{f}(x_n, y_n)$; $x_{n+1}=x_n+h$, with given values for x_0, y_0 and h

3.2.2 FS1: Statistics

Students may use relevant formulae included in the formulae booklet without proof.

FS1.1: Bayes' Theorem

Content	Additional information
Tree diagrams.	Construction, including assigning probabilities, and use in problem solving.
Bayes' Theorem.	Knowledge and application to at most three events.

FS1.2: Uniform distribution

Content	Additional information
Conditions for application.	
Calculation of probabilities.	
Mean and variance.	Knowledge and derivations will be expected.

FS1.3: Geometric distribution

Content	Additional information
Conditions for application.	
Calculation of probabilities.	
Mean and variance.	Knowledge and derivations will be expected.

FS1.4: Probability generating functions (pgf)

Content	Additional information
Definition of a pgf.	$\mathrm{G}_X(t) = \mathrm{E}(t^X) = \sum t^{x_i} p_i$
Properties.	To include: $\mathrm{P}(X = x)$ = coefficient of t^x in $\mathrm{G}_X(t)$, $\mu = \mathrm{G}'_X(1)$ and $\sigma^2 = \mathrm{G}''_X(1) + \mu - \mu^2$
Derivations.	To include, but not exclusively, the uniform, Bernoulli, binomial and geometric distributions.
Sum of independent random variables.	Knowledge and application of the property that the pgf of the sum of independent random variables is the product of their pgfs.

FS1.5: Linear combinations of discrete random variables

Content	Additional information
Mean and variance of a linear combination of two discrete random variables.	To include covariance and correlation. Applications, rather than proofs, will usually be required.
Mean and variance of a linear combination of independent discrete random variables.	Applications, rather than proofs, will usually be required.

3.2.3 FM1: Further mechanics

Students may use relevant formulae included in the formulae booklet without proof.

FM1.1: Constant velocity in two dimensions

Content	Additional information
Displacement, speed, velocity.	
Application of vectors in two dimensions to represent position and velocity.	Vectors may be expressed in terms of the unit vectors **i** and **j** or as column vectors.
Problems involving resultant velocities.	To include solutions using either vectors or vector triangles.
Relative velocity. Use of relative velocity and initial conditions to find relative displacement. Interception and closest approach.	 Geometric approaches may be required. Use of calculus or completing the square.

FM1.2: Dimensional analysis

Content	Additional information
Finding dimensions of quantities. Prediction of formulae. Checks on working, using dimensional consistency.	Finding the dimensions of quantities in terms of M, L and T. Using this method to predict the indices in proposed formulae, for example, for the period of a simple pendulum. Use dimensional analysis to find units, and as a check on working.

FM1.3: Collisions in one dimension

Content	Additional information
Momentum.	
Impulse as change of momentum.	Knowledge and use of the equation $I = mv - mu$
Impulse as Force × Time	$I = Ft$ for a constant force F
Impulse as $\int F \, dt$	For a variable force F
Conservation of momentum.	$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
Newton's Experimental Law. Coefficient of restitution.	$v = eu$ $v_1 - v_2 = -e(u_1 - u_2)$

3.3 International A-level Unit FP2 (Pure maths)

Students will be expected to be familiar with the knowledge, skills and understanding implicit in the pure content of International AS Mathematics and AS Further Mathematics.

Students may use relevant formulae included in the formulae booklet without proof except where proof is required in this unit and requested in a question.

FP2.1: Roots and polynomials

Content	Additional information
The relations between the roots and the coefficients of a polynomial equation; the occurrence of the non-real roots in conjugate pairs when the coefficients of the polynomial are real.	

FP2.2: De Moivre's Theorem

Content	Additional information
De Moivre's theorem for integral n	Use of $z+\frac{1}{z}=2\cos\theta$ and $z-\frac{1}{z}=2\text{ i }\sin\theta$, leading to, for example, expressing $\sin^5\theta$ in terms of multiple angles and $\tan 5\theta$ in term of powers of $\tan\theta$. Applications in evaluating integrals, for example, $\int\sin^5\theta\,\mathrm{d}\theta$
De Moivre's theorem; the *n*th roots of unity, the exponential form of a complex number.	The use, without justification, of the identity $\mathrm{e}^{\mathrm{i}x}=\cos x+\mathrm{i}\sin x$
Solutions of equations of the form $z^n=a+\mathrm{i}b$	To include geometric interpretation and use, for example, in expressing $\cos\frac{5\pi}{12}$ in surd form

FP2.3: Polar coordinates

Content	Additional information
Relationship between polar and Cartesian coordinates.	The convention $r>0$ will be used. The sketching of curves given by equations of the form $r=\mathrm{f}(\theta)$ may be required. Knowledge of the formula $\tan\phi=r\frac{\mathrm{d}\theta}{\mathrm{d}r}$ is not required.
Use of the formula $\text{area}=\int_{\alpha}^{\beta}\frac{1}{2}r^2\,\mathrm{d}\theta$	

FP2.4: Proof by induction

Content	Additional information
Applications to sequences and series, and other problems.	eg proving that 7^n+4^n+1 is divisible by 6, or $(\cos\theta+\mathrm{i}\sin\theta)^n=\cos n\theta+\mathrm{i}\sin n\theta$ where n is a positive integer.

FP2.5: Finite series

Content	Additional information
Summation of a finite series by any method such as induction, partial fractions or differencing.	eg Find the value of $\sum_{r=1}^{n}\frac{1}{(2r-1)(2r+1)}$

FP2.6: Series and limits

Content	Additional information
Maclaurin series	
Expansions of e^x, $\ln(1+x)$, $\cos x$ and $\sin x$, and $(1+x)^n$ for rational values of n	Use of the range of values of x for which these expansions are valid, as given in the formulae booklet, is expected to determine the range of values for which expansions of related functions are valid; eg $\ln\left(\dfrac{1+x}{1-x}\right)$; $(1-2x)^{\frac{1}{2}}e^x$
Knowledge and use, for $k>0$, of $\lim x^k e^{-x}$ as x tends to infinity and $\lim x^k \ln x$ as x tends to zero.	
Improper integrals.	eg $\int_0^e x\ln x \, dx$, $\int_0^\infty xe^{-x} \, dx$ Students will be expected to show the limiting processes used.
Use of series expansion to find limits.	eg $\lim\limits_{x\to 0}\dfrac{e^x-1}{x}$; $\lim\limits_{x\to 0}\dfrac{\sin 3x}{x}$; $\lim\limits_{x\to 0}\dfrac{x^2 e^x}{\cos 2x - 1}$; $\lim\limits_{x\to 0}\dfrac{\sqrt{2+x}-\sqrt{2}}{x}$

FP2.7: The calculus of inverse trigonometrical functions

Content	Additional information
The calculus of inverse trigonometrical functions.	Use of the derivatives of $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$ as given in the formulae booklet. To include the use of the standard integrals $\int \dfrac{1}{a^2+x^2}\,dx$; $\int \dfrac{1}{\sqrt{a^2-x^2}}\,dx$ given in the formulae booklet.

FP2.8: Arc length and area of surface of revolution about the x-axis

Content	Additional information
Calculation of the arc length of a curve and the area of a surface of revolution using Cartesian or parametric coordinates.	Use of the following formulae will be expected: $s = \int_{x_1}^{x_2}\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{\frac{1}{2}} dx \quad = \int_{t_1}^{t_2}\left[\left(\dfrac{dx}{dt}\right)^2+\left(\dfrac{dy}{dt}\right)^2\right]^{\frac{1}{2}} dt$ $S = 2\pi\int_{x_1}^{x_2} y\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{\frac{1}{2}} dx = 2\pi\int_{t_1}^{t_2} y\left[\left(\dfrac{dx}{dt}\right)^2+\left(\dfrac{dy}{dt}\right)^2\right]^{\frac{1}{2}} dt$

FP2.9: Hyperbolic functions

Content	Additional information
Hyperbolic and inverse hyperbolic functions and their derivatives; applications to integration.	The proofs mentioned below require expressing hyperbolic functions in terms of exponential functions. To include solution of equations of the form $a\sinh x + b\cosh x = c$ Use of basic definitions in proving simple identities. Eg Prove the identity: $\sinh(x + y) \equiv \sinh x \cosh y + \cosh x \sinh y$ The logarithmic forms of the inverse functions, given in the formulae booklet, may be required. Proofs of these results may also be required. Proofs of the results of differentiation of the hyperbolic functions, given in the formula booklet, are included. Knowledge, proof and use of: $\cosh^2 x - \sinh^2 x = 1$ $1 - \tanh^2 x = \operatorname{sech}^2 x$ $\coth^2 x - 1 = \operatorname{cosech}^2 x$ Familiarity with the graphs of $\sinh x$, $\cosh x$, $\tanh x$, $\sinh^{-1} x$, $\cosh^{-1} x$, $\tanh^{-1} x$

FP2.10: Differential equations – first order

Content	Additional information
Analytical solution of first order linear differential equations of the form.	
$\frac{dy}{dx} + Py = Q$ where P and Q are functions of x	To include use of an integrating factor and solution by complementary function and particular integral. General solutions and particular solutions using boundary values and initial conditions.

FP2.11: Differential equations – second order

Content	Additional information
Solution of differential equations of the form $a\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+b\frac{\mathrm{d}y}{\mathrm{d}x}+cy=0$, where a, b and c are integers, by using an auxiliary equation whose roots may be real or complex.	General solutions and particular solutions using boundary values and initial conditions. Including repeated roots.
Solution of equations of the form $a\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+b\frac{\mathrm{d}y}{\mathrm{d}x}+cy=\mathrm{f}(x)$ where a, b and c are integers by finding the complementary function and a particular integral.	Finding particular integrals will be restricted to cases where $\mathrm{f}(x)$ is of the form e^{kx}, $\cos kx$, $\sin kx$ or a polynomial of degree at most 4, or a linear combination of any of the above.

FP2.12: Vectors and three-dimensional coordinate geometry

Content	Additional information
Definition and properties of the vector product. Calculation of vector products.	Including the use of vector products in the calculation of the area of a triangle or parallelogram.
Calculation of scalar triple products.	Including the use of the scalar triple product in the calculation of the volume of a parallelepiped and in identifying coplanar vectors.
Applications of vectors to two- and three-dimensional geometry, involving points, lines and planes.	Including the equation of a line in the form $(\mathbf{r}-\mathbf{a})\times\mathbf{b}=0$ Vector equation of a plane in the form $\mathbf{r}.\,\mathbf{n}=d$ or $\mathbf{r}=\mathbf{a}+\lambda\mathbf{b}+\mu\mathbf{c}$ Intersection of a line and a plane. Angle between a line and a plane and between two planes.
Cartesian coordinate geometry of lines and planes. Direction ratios and direction cosines.	To include finding the equation of the line of intersection of two non-parallel planes. Including the use of $l^2+m^2+n^2=1$ where l, m, n are the direction cosines. Knowledge of formulae other than those in the formulae booklet will not be expected.

FP2.13: Matrix algebra

Content	Additional information
Rotations, reflections and enlargements in three dimensions, and combinations of these.	Rotations about the coordinate axes only. Reflections in the planes $x=0,\ y=0,\ z=0,\ x=y,\ x=z,\ y=z$ only.
The determinant of a 3 x 3 matrix.	Including the use of the result $\det(\mathbf{AB})=\det\mathbf{A}\det\mathbf{B}$, but a general treatment of products is not required. Volume scale factor.
The inverse of a 3 x 3 matrix.	
Invariant points and invariant lines in three dimensions.	
Eigenvalues and eigenvectors of 2×2 and 3×3 matrices.	Characteristic equations. Real eigenvalues only. Repeated eigenvalues may be included.

FP2.14: Solution of linear equations

Content	Additional information
Consideration of up to three linear equations in up to three unknowns. Their geometrical interpretation and solution.	Any method of solution is acceptable. Interpretation of configuration of planes.

3.4 International A-level Unit FS2 (Statistics)

Students will be expected to be familiar with the knowledge, skills and understanding implicit in the International AS Mathematics and Further Mathematics.

Students may use relevant formulae included in the formulae booklet without proof.

Electronic calculators or graphical calculators may be used.

FS2.1: Moment generating functions (mgf)

Content	Additional information
Definition of an mgf.	$\mathrm{M}_X(t) = \mathrm{E}\left(\mathrm{e}^{tX}\right) = \sum \mathrm{e}^{tx_i} p_i$ or $\int \mathrm{e}^{tx}\,\mathrm{f}(x)\,\mathrm{d}x$
Properties.	To include: $\mu = \mathrm{M}'_X(0)$ and $\sigma^2 = \mathrm{M}''_X(0) - \mu^2$, $\mathrm{M}_{a+bX}(t) = \mathrm{e}^{at}\,\mathrm{M}_X(bt)$
Derivations.	To include, but not exclusively, the Poisson, exponential and normal.
Sum of independent random variables.	Knowledge and application of the property that the mgf of the sum of independent random variables is the product of their mgfs.

FS2.2: Estimators

Content	Additional information
Review of key concepts.	Sample statistic and its sampling distribution. Population parameter.
Estimators and estimates.	
Properties of estimators.	Unbiasedness, consistency and relative efficiency. Application, including means and variances of pooled estimators.

FS2.3: Estimation

Content	Additional information
Introduction to confidence intervals.	Only confidence intervals symmetrical about the mean will be required.
Confidence intervals for the mean of a distribution.	For a normal distribution with known variance. For large samples using a normal approximation with known or unknown variance. For small samples from a normal distribution with unknown variance.
Inferences from confidence intervals.	Based on whether a constructed confidence interval includes or does not include a hypothesised mean value.
Estimation of sample size.	Necessary to achieve a confidence interval of a required width with a given level of confidence.

FS2.4: Further hypothesis testing

Content	Additional information
Power of a test.	Calculation of P(Type II error) or Power for a simple alternative hypothesis of a test, but not the derivation of a power function. The significance level to be used in a hypothesis test will be given.
Tests for the difference between the means of two independent distributions.	For two normal distributions with known variances. For large samples using normal approximations with known or unknown variances. For small samples from normal distributions with unknown variances, only when the population variances may be assumed equal so that a pooled estimate of variance may be calculated.
Tests for the difference between the means of two non-independent distributions.	For small paired samples using a t-statistic.
Tests for the variance or the standard deviation of a normal distribution.	Using a χ^2-statistic.
Tests for the ratio of the variances or the standard deviations of two independent normal distributions.	Using an F-statistic.
Contingency table and goodness of fit tests.	Use of $\sum \frac{(O_i - E_i)^2}{E_i}$ as an approximate χ^2-statistic. The convention that all E_i should be greater than 5 will be expected. Use of Yates' correction for 2×2 contingency tables will be expected. Goodness of fit tests will be based on a discrete distribution or on a continuous distribution where integration may be required.

3.5 International A-level Unit FM2 (Mechanics)

Students will be expected to be familiar with the knowledge, skills and understanding implicit in the International AS Mathematics and Further Mathematics.

Students may use relevant formulae included in the formulae booklet without proof.

Students should learn the following formulae, which are **not** included in the formulae booklet, but which may be required to answer questions.

Hooke's law	$T = \frac{\lambda}{l} e \ = ke$
Work and energy	Work done, variable force in direction of motion in a straight line: Work $= \int \mathrm{F}\, \mathrm{d}x$ Elastic potential energy $= \frac{\lambda}{2l} e^2 \ = \frac{1}{2} ke^2$
Momentum and collision	$\mathbf{I} = m\mathbf{v} - m\mathbf{u}$; $\mathbf{I} = \mathbf{F}t$ $m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$
Simple harmonic motion	$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -\omega^2 x$ $v^2 = \omega^2(a^2 - x^2)$

FM2.1: Vertical circular motion

Content	Additional information
Circular motion in a vertical plane.	Includes conditions to complete vertical circles.

FM2.2: Projectiles launched onto inclined planes

Content	Additional information
Projectiles launched onto inclined planes.	Problems will be set on projectiles that are launched and land on an inclined plane. Students may approach these problems by resolving the acceleration parallel and perpendicular to the plane. Students will be expected to find the maximum range for a given slope and speed of projection. Students may be expected to determine whether a projectile lands at a higher or lower point on the plane after a bounce.

FM2.3: Elastic strings and springs

Content	Additional information
Knowledge and use of Hooke's law.	$T = \frac{\lambda}{l}e = ke$ Students will be required to be familiar with both modulus of elasticity and stiffness. They should be aware of and understand the relationship $k = \frac{\lambda}{l}$
Work done by a variable force.	Use of $\int F \, \mathrm{d}x$
Elastic potential energy for strings and springs.	Students will be expected to quote the formula for elastic potential energy unless explicitly asked to derive it.

FM2.4: Collisions in two dimensions

Content	Additional information
Momentum as a vector.	
Impulse as a vector.	$\mathbf{I} = m\mathbf{v} - m\mathbf{u}$ and $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$ will be required.
Conservation of momentum in two dimensions.	$m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$
Coefficient of restitution and Newton's experimental law.	
Impacts with a fixed surface.	The impact may be at any angle to the surface. Students may be asked to find the impulse on the body.
Oblique collisions.	Collisions between two smooth spheres. Students will be expected to consider components of velocities parallel and perpendicular to the line of centres.

FM2.5: Application of differential equations

Content	Additional information
Motion in a straight line.	
Problems where differential equations are formed as a result of the application of Newton's second law.	Separable differential equations of the form $\frac{\mathrm{d}v}{\mathrm{d}t} = f(t), \quad \frac{\mathrm{d}v}{\mathrm{d}t} = f(v), \quad \frac{\mathrm{d}v}{\mathrm{d}t} = f(x)$ The result $\frac{\mathrm{d}v}{\mathrm{d}t} = v\frac{\mathrm{d}v}{\mathrm{d}x}$ may be required in questions. Problems may be set where the force of resistance at speed v is of the form $a + bv$ or $a + bv^2$

FM2.6: Simple harmonic motion

Content	Additional information
Knowledge of the definition of simple harmonic motion.	Problems will be set involving elastic strings and springs.
Finding frequency, period and amplitude.	
Knowledge and use of the formula $v^2 = \omega^2\left(a^2 - x^2\right)$	
Formation of simple second order differential equations to show that simple harmonic motion takes place.	
Solution of second order differential equations of the form $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\omega^2 x$	State solutions in the form $x = A\cos\left(\omega t + \alpha\right)$ or $x = A\cos\left(\omega t\right) + B\sin\left(\omega t\right)$ and use these in problems.
Simple pendulum.	Formation and solution of the differential equation, including the use of a small angle approximation. Finding the period.

4 Scheme of assessment

Find mark schemes, and specimen papers for new courses, on our website at **oxfordaqa.com/9665**

These qualifications are modular. The full International A-level is intended to be taken over two years. The specification content for the International AS is half that of an International A-level.

The International AS can be taken as a stand-alone qualification or it can count towards the International A-level. To complete the International A-level, students can take the International AS in their first year and the International A2 in their second year or they can take all the units together in the same examination series at the end of the two year course.

The International AS content will be 50% of the International A-level content. International AS assessments contribute 40% of the total marks for the full International A-level qualification. The remaining 60% comes from the International A2 assessments.

The specification provides an opportunity for students to produce extended responses either in words or using open-ended calculations.

The specification content will be split across units and will include some synoptic assessment. This allows students to draw together different areas of knowledge from across the full course of study.

All materials are available in English only.

4.1 Availability of assessment units and certification

Exams and certification for this specification are available as follows:

	Availability of units		Availability of certification	
	FP1 and FPSM1	**FP2, FS2, FM2**	**International AS**	**International A-level**
June 2018	✓		✓	
January 2019	✓		✓	
June 2019	✓	✓	✓	✓
January 2020	✓	✓	✓	✓
June 2020	✓	✓	✓	✓

4.2 Aims

Courses based on this specification should encourage students to:

- develop their understanding of mathematics and mathematical processes in a way that promotes confidence and fosters enjoyment
- develop abilities to reason logically and to recognise incorrect reasoning, to generalize and to construct mathematical proofs
- extend their range of mathematical skills and techniques and use them in more difficult unstructured problems
- develop an understanding of coherence and progression in mathematics and how different areas of mathematics can be connected
- recognize how a situation may be represented mathematically and understand the relationship between 'real world' problems and standard and other mathematical models and how these can be refined and improved
- use mathematics as an effective means of communication
- read and comprehend mathematical arguments and articles concerning applications of mathematics
- acquire the skills needed to use technology such as calculators and computers effectively yet be aware of any limitations of using these
- develop an awareness of the relevance of mathematics to other fields of study, to the world of work and to society in general.

4.3 Assessment Objectives

The exams will measure how students have achieved the following assessment objectives:

- AO1: Recall and select knowledge of mathematical facts, concepts, models and techniques required to solve problems in a variety of contexts.
- AO2: Construct rigorous mathematical arguments and proofs through use of precise statements, mathematical manipulation, logical deduction, modelling assumptions and justifications to solve structured and unstructured problems, and to deduce, interpret and communicate results.

Quality of Written Communication (QWC)

Students must:

- ensure that text is legible and that spelling, punctuation and grammar are accurate so that meaning is clear
- select and use a form and style of writing appropriate to purpose and to complex subject matter
- organise information clearly and coherently, using specialist vocabulary when appropriate.

Questions in the papers for this specification do not include specific marks for QWC. However, poor written communication may lead to lower marks due to lack of clarity in answers.

4.3.1 Assessment Objective weightings for International AS Further Mathematics

Assessment Objectives (AOs)	Unit weightings (approx %)		Overall weighting of AOs (approx %)
	Unit FP1	Unit FPSM1	
AO1	20–25	20–25	40–50
AO2	25–30	25–30	50–60
Overall weighting of units (%)	50	50	100

4.3.2 Assessment Objective weightings for International A-level Further Mathematics

Assessment Objectives (AOs)	Unit weightings (approx %)				Overall weighting of AOs (approx %)
	Unit FP1	Unit FPSM1	Unit FP2	Unit FS2 or FM2	
AO1	8–10	8–10	15–18.75	9–11.25	40–50
AO2	10–12	10–12	18.75–22.5	11.25–13.5	50–60
Overall weighting of units (%)	20	20	37.5	22.5	100

4.4 Assessment weightings

The raw marks awarded on each unit will be transferred to a uniform mark scale (UMS) to meet the weighting of the units and to ensure comparability between units sat in different exam series. Students' final grades will be calculated by adding together the uniform marks for all units. The maximum raw and uniform marks are shown in the table below.

Unit	**Maximum raw mark**	**Percentage weighting A-level (AS)**	**Maximum uniform mark**
FP1	80	20 (50)	80
FPSM1	80	20 (50)	80
FP2	120	37.5	150
FS2	80	22.5	90
FM2	80	22.5	90
Qualification			
International AS (FP1 + FPSM1)	–	40 (100)	160
International A-level (FP1 + FPSM1 + FP2 + FS2 or FM2)	–	100	400

For more detail on UMS, see Section 5.3.

5 General administration

We are committed to delivering assessments of the highest quality and have developed practices and procedures to support this aim. To ensure all students have a fair experience, we have worked with other awarding bodies in England to develop best practice for maintaining the integrity of exams. This is published through the Joint Council for Qualifications (JCQ). We will maintain the same high standard through their use for OxfordAQA Exams.

More information on all aspects of administration is available at **oxfordaqa.com/exams-administration**

For any immediate enquiries please contact **info@oxfordaqa.com**

Please note: We aim to respond to all email enquiries within two working days.

Our UK office hours are Monday to Friday, 8am – 5pm local time.

5.1 Entries and codes

You should use the following subject award entry codes:

Qualification title	OxfordAQA Exams entry code
OxfordAQA International Advanced Subsidiary Further Mathematics	9666
OxfordAQA International Advanced Level Further Mathematics	9667

Please check the current version of the Entry Codes book and the latest information about making entries on **oxfordaqa.com/exams-administration**

You should use the following unit entry codes:

Unit FP1 – FM01

Unit FPSM1 – FM02

Unit FP2 – FM03

Unit FS2 – FM04

Unit FM2 – FM05

A unit entry will not trigger certification. You will also need to make an entry for the overall subject award in the series that certification is required.

Exams will be available May/June and in January.

5.2 Overlaps with other qualifications

There is overlapping content in the International AS and A-level specifications. This helps you teach the International AS and A-level together.

5.3 Awarding grades and reporting results

The International AS qualification will be graded on a five-point scale: A, B, C, D and E.

The International A-level qualification will be graded on a six-point scale: A*, A, B, C, D and E.

Students who fail to reach the minimum standard for grade E will be recorded as U (unclassified) and will not receive a qualification certificate.

We will publish the minimum raw mark needed for each grade in each unit when we issue students' results. We will report a student's unit results to schools in terms of uniform marks and unit grades and we will report qualification results in terms of uniform marks and grades.

The relationship between uniform marks and grades is shown in the table below.

	Uniform mark range per unit and per qualification						
Grade	FP1	FPSM1	International AS Further Mathematics	FP2	Option FM2	Option FS2	International A-level Further Mathematics
Maximum uniform mark	80	80	160	150	90	90	400
A*				135–150	81–90	81–90	344*
A	64–80	64–80	128–160	120–134	72–80	72–80	320
B	56–63	56–63	112–127	105–119	63–71	63–71	280
C	48–55	48–55	96–111	90–104	54–62	54–62	240
D	40–47	40–47	80–95	75–89	45–53	45–53	200
E	32–39	32–39	64–79	60–74	36–44	36–44	160

* For the award of grade A*, a student must achieve grade A in the full International A-level qualification and a minimum of 216 uniform marks in the aggregate of units FP2 with FM2 or FS2.

5.4 Re-sits

Unit results remain available to count towards certification, whether or not they have already been used, provided the specification remains valid. Students can re-sit units as many times as they like, as long as they're within the shelf-life of the specification. The best result from each unit will count towards the final qualification grade. Students who wish to repeat a qualification may do so by re-sitting one or more units.

To be awarded a new subject grade, the appropriate subject award entry, as well as the unit entry/entries, must be submitted.

5.5 Previous learning and prerequisites

There are no previous learning requirements. Any requirements for entry to a course based on this specification are at the discretion of schools.

5.6 Access to assessment: equality and inclusion

Our general qualifications are designed to prepare students for a wide range of occupations and further study whilst assessing a wide range of competences.

The subject criteria have been assessed to ensure they test specific competences. The skills or knowledge required do not disadvantage particular groups of students.

Exam access arrangements are available for students with disabilities and special educational needs.

We comply with the *UK Equality Act 2010* to make reasonable adjustments to remove or lessen any disadvantage that affects a disabled student. Information about access arrangements is issued to schools when they become OxfordAQA centres.

5.7 Working with OxfordAQA for the first time

You will need to apply to become an OxfordAQA centre to offer our specifications to your students. Find out how at **oxfordaqa.com/centreapprovals**

5.8 Private candidates

Centres may accept private candidates for examined units/components only with the prior agreement of OxfordAQA. If you are an approved OxfordAQA centre and wish to accept private candidates, please contact OxfordAQA at: **info@oxfordaqa.com**

Private candidates may also enter for examined only units/components via the British Council; please contact your local British Council office for details.

OXFORD AQA
INTERNATIONAL QUALIFICATIONS

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Thank you for choosing OxfordAQA, the international exam board that puts fairness first.

Benchmarked to UK standards, our exams only ever test subject ability, not language skills or cultural knowledge.

This gives every student the best possible chance to show what they can do and get the results they deserve.

Get in touch

You can contact us at *oxfordaqa.com/contact-us*
or email *info@oxfordaqa.com*

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