OXFORD AQA
INTERNATIONAL QUALIFICATIONS

# INTERNATIONAL A-LEVEL
# FURTHER MATHEMATICS
# FM5P3

Further Pure Mathematics 3

Mark scheme

Specimen

Version: 1.0

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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**Key to mark scheme abbreviations**

| | |
|---|---|
| **M** | Mark is for method |
| **m** | Mark is dependent on one or more M marks and is for method |
| **A** | Mark is dependent on M or m marks and is for accuracy |
| **B** | Mark is independent of M or m marks and is for method and accuracy |
| **E** | Mark is for explanation |
| **ft** | Follow through from previous incorrect result |
| **CAO** | Correct answer only |
| **CSO** | Correct solution only |
| **AWFW** | Anything which falls within |
| **AWRT** | Anything which rounds to |
| **ACF** | Any correct form |
| **AG** | Answer given |
| **SC** | Special case |
| **oe** | Or equivalent |
| **NMS** | No method shown |
| **PI** | Possibly implied |
| **sf** | Significant figure(s) |
| **dp** | Decimal place(s) |
| **ISW** | Ignore subsequent working |

**Mark Scheme Alignment**

Marks in the Marks column will typically align with the relevant line of mathematics in the Answer column where the mark is awarded. If two or more marks are awarded for the same line of mathematics, then the Guidance column will give further details as to how each mark is awarded.

**Square Brackets**

Where square brackets appear in a line of the mark scheme where one or more marks are awarded, the mathematics inside the square bracket is not required for marks to be awarded. Instead, they are intended to be helpful to users of the mark scheme.

Exceptions to this include integration, vectors and matrices where square brackets carry a different mathematical meaning.

| Q | Answer | Marks | Comments |
|---|---|---|---|
| 1 | $e^{4x} = 1 + 4x + 8x^2 + \ldots$ | B1 | Seen or used |
| | $(1+8x)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)(8x) + \frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}(8x)^2 + \ldots$ | M1 | Unsimplified binomial expansion |
| | $(1+8x)^{-\frac{1}{2}} = 1 - 4x + 24x^2 + \ldots$ | A1 | Fully correct and simplified binomial expansion<br>**PI** |
| | $e^{4x}(1+8x)^{-\frac{1}{2}}$<br>$= (1 + 4x + 8x^2 + \ldots)\times(1 - 4x + 24x^2 + \ldots)$ | | |
| | $x^2$ term: | | |
| | $24x^2 - 16x^2 + 8x^2 = 16x^2$ | M1 | |
| | $k = 16$ | A1 | |
| | **Subtotal** | **5** | |

| | | | |
|---|---|---|---|
| | **Question 1 Total** | **5** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| **2** | $\frac{\mathrm{d}y}{\mathrm{d}x} = 12\cosh x \sinh x + 5\cosh x$ | **B1** | Correct derivative |
| | $\cosh x\left(12\sinh x + 5\right) = 0$ | **M1** | Sets their derivative equation to 0 |
| | $\cosh x \neq 0$ as $\cosh x \geq 1$ | **E1** | |
| | Stationary points occur when $\sinh x = -\frac{5}{12}$, which has one solution. Hence, *D* has one stationary point | **A1** | **AG** Must be convincingly shown<br>**B1 M1 E0 A1** is possible |
| | **Subtotal** | **4** | |

| | | | |
|---|---|---|---|
| | **Question 2 Total** | **4** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| 3(a) | $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{1+\left(\sqrt{3x}\right)^2} \times \frac{1}{2} \times \sqrt{3} \times \frac{1}{\sqrt{x}}$ | M1 | oe |
| | $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sqrt{3}}{2\sqrt{x}\left(1+3x\right)}$ | A1 | Correct simplified derivative |
| | **Subtotal** | **2** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| 3(b) | $\int_{\frac{1}{3}}^{1} \frac{1}{\left(1+3x\right)\sqrt{x}}\,\mathrm{d}x$ | | |
| | $= \left[\frac{2}{\sqrt{3}}\tan^{-1}\left(\sqrt{3x}\right)\right]_{\frac{1}{3}}^{1}$ | B1ft | Correct integration<br>**ft** their $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{k}{\sqrt{x}\left(1+3x\right)}$ from part (a)<br>where $k \neq 0,1$ |
| | $= \frac{2}{\sqrt{3}}\tan^{-1}\left(\sqrt{3}\right) - \frac{2}{\sqrt{3}}\tan^{-1}\left(1\right)$ | | |
| | $= \frac{2\pi}{3\sqrt{3}} - \frac{\pi}{2\sqrt{3}}$ | M1 | Substitutes limits in to their integration |
| | $= \frac{\pi\sqrt{3}}{18}$ | A1 | **CAO** |
| | **Subtotal** | **3** | |

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|---|---|---|---|
| | **Question 3 Total** | **5** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| **4(a)** | $\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{2x}{x^2+1}y = 3-4x$<br><br>Integrating factor is:<br><br>$\mathrm{IF} = \exp\left(\int \frac{2x}{x^2+1}\,\mathrm{d}x\right) = \exp\left(\ln\left(x^2+1\right)\right)$ | **M1** | Integrating factor identified and integration attempted |
| | $\mathrm{IF} = x^2+1$ | **A1** | Correct integrating factor |
| | $\left(x^2+1\right)\frac{\mathrm{d}y}{\mathrm{d}x} + 2xy = \left(3-4x\right)\left(x^2+1\right)$<br><br>$\left(x^2+1\right)y = \int \left(3-4x\right)\left(x^2+1\right)\mathrm{d}x$ | **m1** | Multiplying both sides of the differential equation by their integrating factor and then integrating LHS to get $\mathrm{IF}\times y$ |
| | $\left(x^2+1\right)y = -x^4+x^3-2x^2+3x+c$ | **M1** | Integrates RHS |
| | $\mathrm{f}\left(x\right) = \frac{-x^4+x^3-2x^2+3x+c}{x^2+1}$ | **A1** | Correct general solution |
| | **Subtotal** | **5** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| **4(b)** | $\mathrm{f}\left(1\right) = \frac{-1+1-2+3+c}{1+1} = 9$ | **M1** | Substitutes $x=1$ into their general solution and sets equal to 9<br>Must have a constant of integration in the equation |
| | $c = 17$<br><br>$\mathrm{f}\left(x\right) = \frac{-x^4+x^3-2x^2+3x+17}{x^2+1}$ | **A1** | Correct particular solution |
| | **Subtotal** | **2** | |

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|---|---|---|---|
| | **Question 4 Total** | **7** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| **5(a)** | $\sin(2x) = 2x - \frac{(2x)^3}{3!} + \ldots$<br><br>$\frac{2x - \frac{(2x)^3}{3!} + \ldots}{x}$<br><br>$= 2 - \frac{4x^2}{3} + \ldots$<br><br>$\lim_{x \to 0}\left(2 - \frac{4x^2}{3} + \ldots\right) = 2$ | **M1**<br><br>**A1** | Maclaurin series expansion for $\sin(2x)$ in numerator and $x$ in denominator<br><br>**AG** Must be convincingly shown with all intermediate working present |
| | **Subtotal** | **2** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| **5(b)(i)** | $2\cot(2x)$ and $\frac{1}{x}$ are undefined at $x = 0$<br>[which is in the interval of integration] | **E1** | Correct explanation |
| | **Subtotal** | **1** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| **5(b)(ii)** | $\int_0^{\frac{\pi}{4}} \left(2\cot(2x) - \frac{1}{x}\right) \mathrm{d}x$ | | |
| | $\lim_{a\to 0} \int_0^{a} \left(2\cot(2x) - \frac{1}{x}\right) \mathrm{d}x$ | **M1** | Lower limit replaced with $a$ **oe** and $\lim_{a\to 0}$ seen at any stage in the solution |
| | $\lim_{a\to 0} \left[\ln(\sin 2x) - \ln x\right]_a^{\frac{\pi}{4}}$ | **A1** | Correct integration |
| | $= \left(\ln\left(\sin\left(\frac{\pi}{2}\right)\right) - \ln\left(\frac{\pi}{4}\right)\right)$ | | |
| | $- \lim_{a\to 0} \left(\ln(\sin(2a)) - \ln(a)\right)$ | | |
| | $= -\ln\left(\frac{\pi}{4}\right) - \lim_{a\to 0} \ln\left(\frac{\sin(2a)}{a}\right)$ | **M1** | Substitutes in the limits and evaluates all terms |
| | $= -\ln\left(\frac{\pi}{4}\right) - \ln(2)$ | | |
| | $= -\ln\left(\frac{\pi}{2}\right)$ | **A1** | **ACF** such as $= \ln\left(\frac{2}{\pi}\right)$ |
| | **Subtotal** | **4** | |

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|---|---|---|---|
| | **Question 5 Total** | **10** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| 6 | $m^2+4m+5=0 \Rightarrow m=-2+\mathrm{i},\ -2-\mathrm{i}$ | M1 | Forms and solves the correct auxiliary equation |
| | $y_{\mathrm{CF}}=\mathrm{e}^{-2x}\left(A\cos x+B\sin x\right)$ | A1 | Correct complementary function |
| | $y_{\mathrm{PI}}=a\mathrm{e}^{x}+bx^2+cx+d$ | B1 B1 | **B1**: $a\mathrm{e}^{x}$<br>**B1**: $bx^2+cx+d$ |
| | $y_{\mathrm{PI-1}}=a\mathrm{e}^{x},\ y'_{\mathrm{PI-1}}=a\mathrm{e}^{x},\ y''_{\mathrm{PI-1}}=a\mathrm{e}^{x}$ | | |
| | $a\mathrm{e}^{x}+4a\mathrm{e}^{x}+5a\mathrm{e}^{x}=30a\mathrm{e}^{x}$ | M1 | Their $y'_{\mathrm{PI-1}}$ and $y''_{\mathrm{PI-1}}$ or their $y'_{\mathrm{PI-2}}$ and $y''_{\mathrm{PI-2}}$ substituted into the differential equation |
| | $a=3$ | | |
| | $y_{\mathrm{PI-2}}=bx^2+cx+d,\ y'_{\mathrm{PI-2}}=2bx+c,$<br>$y''_{\mathrm{PI-2}}=2b$ | | |
| | $2b+4\left(2bx+c\right)+5\left(bx^2+cx+d\right)$<br>$=10x^2+21x+3$ | | |
| | $5bx^2+\left(8b+5c\right)x+\left(2b+4c+5d\right)$<br>$=10x^2+21x+3$ | | |
| | $5b=10 \Rightarrow b=2$ | M1 | Equates coefficients for their $y_{\mathrm{PI-2}}$ |
| | $8b+5c=21 \Rightarrow c=1$ | | |
| | $2b+4c+5d=3 \Rightarrow d=-1$ | | |
| | $y_{\mathrm{PI}}=3\mathrm{e}^{x}+2x^2+x-1$ | A1 A1 | **A1**: At least two coefficients in the particular integral correct<br>**A1**: Fully correct particular integral |
| | $y_{\mathrm{GS}}=\mathrm{e}^{-2x}\left(A\cos x+B\sin x\right)+3\mathrm{e}^{x}$<br>$+2x^2+x-1$ | B1ft | Their complementary function + their particular integral with exactly two arbitrary constants |
| | When $x=0$,<br>$y=5 \Rightarrow y_{\mathrm{GS}}=A+3-1=5 \Rightarrow A=3$ | A1 | Either $A=3$ or $B=4$ |
| | When $x=0$, | | |

| | | | |
|---|---|---|---|
| | $\frac{dy}{dx} = 2 \Rightarrow \frac{dy_{GS}}{dx} = -2A + B + 3 + 1 = 2$<br>$\Rightarrow B = 4$<br><br>$f(x) = e^{-2x}(3\cos x + 4\sin x) + 3e^{x}$<br>$+ 2x^2 + x - 1$ | **A1** | Fully correct $f(x)$ |
| | **Subtotal** | **11** | |

| | | | |
|---|---|---|---|
| | **Question 6 Total** | **11** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| 7(a) | $y = 2\sqrt{x-1} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{\sqrt{x-1}}$ | B1 | Correct derivative |
| | $1+\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = 1+\dfrac{1}{x-1}$ | | |
| | $1+\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = \dfrac{x}{x-1}$ | M1 | **ft** their $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ |
| | $S = \displaystyle\int_{[2]}^{[9]} \sqrt{1+\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x$ | | |
| | $S = \displaystyle\int_{2}^{9} \sqrt{\dfrac{x}{x-1}}\,\mathrm{d}x$ | A1 | **AG** Must be convincingly shown |
| | **Subtotal** | **3** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| 7(b) | $\cosh^{-1}(3) = \ln\left(3+\sqrt{8}\right)$ | M1 | |
| | $\left(1+\sqrt{2}\right)^2 = 3+2\sqrt{2}$ | | |
| | $\left(1+\sqrt{2}\right)^2 = 3+\sqrt{8}$ | | |
| | $\cosh^{-1}(3) = \ln\left(1+\sqrt{2}\right)^2$ | | |
| | $\cosh^{-1}(3) = 2\ln\left(1+\sqrt{2}\right)$ | A1 | **AG** Must be convincingly shown |
| | **Subtotal** | **2** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| 7(c) | $x = \cosh^2\theta \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}\theta} = 2\cosh\theta\sinh\theta$ | B1 | Correct derivative |
| | $\displaystyle\int \sqrt{\frac{\cosh^2\theta}{\cosh^2\theta - 1}}\, 2\cosh\theta\sinh\theta\,\mathrm{d}\theta$ | M1 | |
| | $= \displaystyle\int \frac{\cosh\theta}{\sinh\theta} \times 2\cosh\theta\sinh\theta\,\mathrm{d}\theta$ | | |
| | $= \displaystyle\int 2\cosh^2\theta\,\mathrm{d}\theta$ | | |
| | $= \displaystyle\int \left(1 + \cosh 2\theta\right)\mathrm{d}\theta$ | A1 | Simplifies integrand using double angle formula or $\frac{1}{2}\left(\mathrm{e}^{2\theta} + 2 + \mathrm{e}^{-2\theta}\right)$ |
| | $= \theta + \frac{1}{2}\sinh 2\theta$ | A1 | **oe**, such as $\frac{1}{4}\mathrm{e}^{2\theta} + \theta - \frac{1}{4}\mathrm{e}^{-2\theta}$ |
| | $S = \left[\theta + \frac{1}{2}\sinh 2\theta\right]_{\cosh^{-1}\left(\sqrt{2}\right)}^{\cosh^{-1}(3)}$ | | |
| | $= \left(\cosh^{-1}(3) + \frac{1}{2}\sinh\left(2\cosh^{-1}(3)\right)\right)$ $-\left(\cosh^{-1}\left(\sqrt{2}\right) + \frac{1}{2}\sinh\left(2\cosh^{-1}\left(\sqrt{2}\right)\right)\right)$ | m1 | Correctly substitutes correctly limits into their integration |
| | $= 2\ln\left(1+\sqrt{2}\right) - \ln\left(1+\sqrt{2}\right) + 6\sqrt{2} - \sqrt{2}$ | | |
| | $= 5\sqrt{2} + \ln\left(1+\sqrt{2}\right)$ | A1 | **CAO** |
| | **Subtotal** | **6** | |

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|---|---|---|---|
| | **Question 7 Total** | **11** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| 8(a)(i) | $x^2 + y^2 = x^2 + 16 - 8x$<br>$x^2 + y^2 = (4 - x)^2$ | B1 | CAO |
| | Subtotal | 1 | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| 8(a)(ii) | $r^2 = (4 - x)^2$ | M1 | $x^2 + y^2 = r^2$ used **oe** |
| | $r^2 = (4 - r\cos\theta)^2$ | m1 | $x = r\cos\theta$ used |
| | $r = 4 - r\cos\theta$ | | |
| | $r = 4 - r\left(2\cos^2\left(\frac{\theta}{2}\right) - 1\right)$ | m1 | Uses double angle formula |
| | $2r\cos^2\left(\frac{\theta}{2}\right) = 4$ | | |
| | $r = 2\sec^2\left(\frac{\theta}{2}\right)$ | A1 | **AG** Must be convincingly shown |
| | Subtotal | 4 | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| 8(b) | $A = \frac{1}{2}\int_{\alpha}^{\beta}\left(2\sec^2\left(\frac{\theta}{2}\right)\right)^2 \mathrm{d}\theta$ | B1 | Fully correct integral with correct limits of integration |
| | $= 2\int_{[\alpha]}^{[\beta]}\sec^4\left(\frac{\theta}{2}\right)\mathrm{d}\theta$ | | |
| | $= 2\int_{[\alpha]}^{[\beta]}\sec^2\left(\frac{\theta}{2}\right)\left(1+\tan^2\left(\frac{\theta}{2}\right)\right)\mathrm{d}\theta$ | M1 | Use of $\sec^4\left(\frac{\theta}{2}\right) = \sec^2\left(\frac{\theta}{2}\right)\left(1+\tan^2\left(\frac{\theta}{2}\right)\right)$ |
| | Let $u = \tan\left(\frac{\theta}{2}\right) \Rightarrow \mathrm{d}u = \frac{1}{2}\sec^2\left(\frac{\theta}{2}\right)$ | | |
| | $\Rightarrow 4\int \left(1+u^2\right)\mathrm{d}u$ | | |
| | $= 4\left[\tan\left(\frac{\theta}{2}\right)+\frac{1}{3}\tan^3\left(\frac{\theta}{2}\right)\right]_{[\alpha]}^{[\beta]}$ | m1 A1 | **m1**: $a\tan\left(\frac{\theta}{2}\right)+b\tan^3\left(\frac{\theta}{2}\right)$<br>**A1**: $4\tan\left(\frac{\theta}{2}\right)+\frac{4}{3}\tan^3\left(\frac{\theta}{2}\right)$ **oe** |
| | $= \left(4\tan\left(\frac{\beta}{2}\right)+\frac{4}{3}\tan^3\left(\frac{\beta}{2}\right)\right)$<br>$-\left(4\tan\left(\frac{\alpha}{2}\right)+\frac{4}{3}\tan^3\left(\frac{\alpha}{2}\right)\right)$ | | |
| | $= 4\tan\left(\frac{\beta}{2}\right)-4\tan\left(\frac{\alpha}{2}\right)$<br>$+\frac{4}{3}\tan^3\left(\frac{\beta}{2}\right)-\frac{4}{3}\tan^3\left(\frac{\alpha}{2}\right)$ | A1 | **ACF** |
| | **Subtotal** | **5** | |

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|---|---|---|---|
| | **Question 8 Total** | **10** | |