OXFORD AQA
INTERNATIONAL QUALIFICATIONS

INTERNATIONAL A LEVEL
FURTHER MATHEMATICS

Further Pure Mathematics 3

Specimen paper 0X:00 GMT Time allowed: 1 hour 15 minutes

Materials
- For this paper you must have the OxfordAQA Pure Mathematics Examination Booklet (enclosed)
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.
- The final answer to questions requiring an algebraic expression should be given in simplified form, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 60.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise, marks may not be awarded.

Copyright © 2026 OxfordAQA International Qualifications and its licensors. All rights reserved.

Answer **all** questions in the spaces provided.

1 It is given that the expansion $e^{4x}(1+8x)^{-\frac{1}{2}}$ in ascending powers of x up to and including the term in x^2 is $1+kx^2$ where k is a constant.

Find the value of k

[5 marks]

$k =$ _____

Copyright © 2026 OxfordAQA International Qualifications and its licensors. All rights reserved.

2 The curve D has equation $y = 6\cosh^2 x + 5\sinh x$

Show that D has exactly one stationary point.

Fully justify your answer.

[4 marks]

Turn over for the next question

Copyright © 2026 OxfordAQA International Qualifications and its licensors. All rights reserved.

3 It is given that $y = \tan^{-1}\left(\sqrt{3x}\right)$

3 (a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$

Give your answer in terms of x

[2 marks]

$\frac{\mathrm{d}y}{\mathrm{d}x} =$ _____

Copyright © 2026 OxfordAQA International Qualifications and its licensors. All rights reserved.

3 (b) Use your answer to **part (a)** to find $\int_{\frac{1}{3}}^{1} \frac{1}{(1+3x)\sqrt{x}}\,\mathrm{d}x$

Give your answer in the form $\frac{\pi\sqrt{p}}{n}$ where p is prime and n is an integer.

[3 marks]

Answer _____

Copyright © 2026 OxfordAQA International Qualifications and its licensors. All rights reserved.

4 The general solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{2x}{x^2+1}y = 3 - 4x$$

can be written in the form $y = \mathrm{f}(x)$

4 (a) Use an integrating factor to find $\mathrm{f}(x)$

[5 marks]

$\mathrm{f}(x) =$ _____

Copyright © 2026 OxfordAQA International Qualifications and its licensors. All rights reserved.

4 (b) Find the particular solution of the differential equation for which $\mathrm{f}(1) = 9$

[2 marks]

$y =$ _____

Turn over for the next question

Copyright © 2026 OxfordAQA International Qualifications and its licensors. All rights reserved.

5 (a) Use the Maclaurin series expansion for $\sin(2x)$ to show that

$$\lim_{x \to 0}\left(\frac{\sin(2x)}{x}\right) = 2$$

[2 marks]

5 (b) (i) Explain why $\int_0^{\frac{\pi}{4}} \left(2\cot(2x) - \frac{1}{x}\right) \mathrm{d}x$ is an improper integral.

[1 mark]

Copyright © 2026 OxfordAQA International Qualifications and its licensors. All rights reserved.

5 (b) (ii) Use the result given in **part (a)** to evaluate

$$\int_0^{\frac{\pi}{4}} \left(2\cot(2x) - \frac{1}{x}\right) \mathrm{d}x$$

Show the limiting process used.

Give your answer in an exact logarithmic form.

[4 marks]

Answer _____

Copyright © 2026 OxfordAQA International Qualifications and its licensors. All rights reserved.

6 It is given that $y = \mathrm{f}(x)$ satisfies the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 4\frac{\mathrm{d}y}{\mathrm{d}x} + 5y = 30\mathrm{e}^x + 10x^2 + 21x + 3$$

It is also given that when $x = 0$ both $y = 5$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 2$

Find $\mathrm{f}(x)$

[11 marks]

Copyright © 2026 OxfordAQA International Qualifications and its licensors. All rights reserved.

$f(x) =$

Turn over for the next question

Copyright © 2026 OxfordAQA International Qualifications and its licensors. All rights reserved.

7 The curve C has equation $y = 2\sqrt{x-1}$ for $x > 1$

The length of the arc of C between the points on C where $x = 2$ and $x = 9$ is S

7 (a) Show that

$$S = \int_2^9 \sqrt{\frac{x}{x-1}}\,\mathrm{d}x$$

[3 marks]

7 (b) Show that $\cosh^{-1}(3) = 2\ln\left(1+\sqrt{2}\right)$

[2 marks]

Copyright © 2026 OxfordAQA International Qualifications and its licensors. All rights reserved.

7 (c) Use the substitution $x = \cosh^2\theta$ and the results in **part (a)** and **part (b)** to find S

Give your answer in the form $n\sqrt{2} + \ln p$ where n is an integer and p is irrational.

[6 marks]

Answer _____

Copyright © 2026 OxfordAQA International Qualifications and its licensors. All rights reserved.

8 The curve C has Cartesian equation

$$y^2 = 8(2 - x) \quad \text{for} \quad x \leq 2$$

8 (a) (i) Express the Cartesian equation of C in the form

$$x^2 + y^2 = (k - x)^2$$

where k is an integer.

[1 mark]

Answer _____

8 (a) (ii) It is given that the origin O is the pole and the positive x-axis is the initial line.

Show that the polar equation of C is

$$r = 2\sec^2\left(\frac{\theta}{2}\right) \quad \text{for} \quad -\pi < \theta < \pi \quad \text{and} \quad r \geq 2$$

[4 marks]

Copyright © 2026 OxfordAQA International Qualifications and its licensors. All rights reserved.

8 (b) The area of the finite region bounded by C, the line $\theta = \alpha$ and the line $\theta = \beta$ is A square units, where $0 < \alpha < \beta < \frac{\pi}{2}$

Find an expression for A

Give your answer in an exact form in terms of $\tan\left(\frac{\alpha}{2}\right)$ and $\tan\left(\frac{\beta}{2}\right)$

[5 marks]

END OF QUESTIONS

Permission to reproduce all copyright material has been applied for. In some cases, efforts to contact copyright-holders may have been unsuccessful and Oxford AQA International Qualifications will be happy to rectify any omissions of acknowledgements. If you have any queries please contact the Copyright Team, AQA, Stag Hill House, Guildford, GU2 7XJ.

Copyright © 2026 OxfordAQA International Qualifications and its licensors. All rights reserved.