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INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS
FM6P4

Further Pure Mathematics 4

Mark scheme

Specimen

Version: 1.0

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
NMS	No method shown
PI	Possibly implied
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Mark Scheme Alignment

Marks in the Marks column will typically align with the relevant line of mathematics in the Answer column where the mark is awarded. If two or more marks are awarded for the same line of mathematics, then the Guidance column will give further details as to how each mark is awarded.

Square Brackets

Where square brackets appear in a line of the mark scheme where one or more marks are awarded, the mathematics inside the square bracket is not required for marks to be awarded. Instead, they are intended to be helpful to users of the mark scheme.

Exceptions to this include integration, vectors and matrices where square brackets carry a different mathematical meaning.

Q	Answer	Marks	Guidance
1(a)	Rotation about x-axis	B1	Both 'Rotation' and 'x-axis'
	$\cos\theta = -0.6$ and $\sin\theta = 0.8$	M1	Relates $\cos\theta$ and $\sin\theta$ to elements of matrix. Condone sign errors
	$\tan\theta = -\frac{4}{3}$		
	$\theta = 127°$		
	through an angle of 127° [anticlockwise]	A1	**AWRT** 127°
	Subtotal	**3**	

Q	Answer	Marks	Guidance
1(b)	$\det\mathbf{P} = 4(k-0) - (-k)(2k+9) + 2(0+3)$	M1	Determinant of **P** with at least two of the three terms correct **PI**
	$\det\mathbf{P} = 2k^2 + 13k + 6$	A1	Correct, simplified determinant of **P**
	$2k^2 + 13k + 6 = 0$		
	$k = -6, -\frac{1}{2}$	A1	Correct critical values for k
	$k < -6$ or $k > -\frac{1}{2}$	A1	Correct range of values for k
	Subtotal	**4**	

	Question 1 Total	**7**	

Q	Answer	Marks	Guidance
2(a)	$\left[\mathbf{a}\bullet\left(\mathbf{b}\times\mathbf{c}\right)=\right]\begin{vmatrix}4 & -28 & p\\ 5 & 3 & -2\\ 2 & -2 & 2\end{vmatrix}=0$	M1	Sets the scalar triple product equal to zero
	$4\begin{vmatrix}3 & -2\\ -2 & 2\end{vmatrix}-\left(-28\right)\begin{vmatrix}5 & -2\\ 2 & 2\end{vmatrix}+p\begin{vmatrix}5 & 3\\ 2 & -2\end{vmatrix}=0$		
	$4\left(6-4\right)+28\left(10+4\right)+p\left(-10-6\right)=0$	m1	Forms a linear equation in p
	$16p=400$		
	$p=25$	A1	CAO
	Subtotal	**3**	

Q	Answer	Marks	Guidance
2(b)	$\mathbf{b}\times\mathbf{c}=\begin{bmatrix}2\\ -14\\ -16\end{bmatrix}$	B1	Finds $\mathbf{b}\times\mathbf{c}$
	$\lambda\begin{bmatrix}2\\ -14\\ -16\end{bmatrix}=\begin{bmatrix}4\\ -28\\ p\end{bmatrix}$	M1	Relates their $\mathbf{b}\times\mathbf{c}$ as being parallel to $\mathbf{a}$
	$\lambda=2$		
	$p=-32$	A1	CAO
	Subtotal	**3**	

	Question 2 Total	**6**	

Q	Answer	Marks	Guidance
3(a)	$\text{Im}\left(\text{e}^{3\text{i}x}\right) = \sin(3x)$ $\Rightarrow I = \int \text{e}^{2x}\,\text{Im}\left(\text{e}^{3\text{i}x}\right)\,\text{d}x$ $I = \int \text{Im}\left(\text{e}^{2x+3\text{i}x}\right)\,\text{d}x$ $I = \text{Im}\left(\int \text{e}^{(2+3\text{i})x}\,\text{d}x\right)$	B1	**AG** Must be convincingly shown with $\text{Im}\left(\text{e}^{3\text{i}x}\right) = \sin(3x)$ clearly used
	Subtotal	**1**	

Q	Answer	Marks	Guidance
3(b)	$\int \text{e}^{(2+3\text{i})x}\,\text{d}x = \frac{1}{2+3\text{i}}\text{e}^{(2+3\text{i})x}\ [+c]$	B1	Correct integration, unsimplified or better
	$= \frac{2-3\text{i}}{13}\text{e}^{2x}\left(\cos(3x)+\text{i}\sin(3x)\right)$	M1 M1	**M1**: $= \frac{2-3\text{i}}{13}\times\ldots$ **M1**: $=\ldots\times\text{e}^{2x}\left(\cos(3x)+\text{i}\sin(3x)\right)$
	$= \frac{\text{e}^{2x}}{13}\left(2\cos(3x)+3\sin(3x)\right)$ $+\text{i}\frac{\text{e}^{2x}}{13}\left(2\sin(3x)-3\cos(3x)\right)$		May not see real part
	$I = \frac{\text{e}^{2x}}{13}\left(2\sin(3x)-3\cos(3x)\right)\ [+c]$	A1	Takes imaginary part and obtains the correct final answer
	Subtotal	**4**	

	Question 3 Total	**5**	

Q	Answer	Marks	Guidance
4(a)	$\begin{vmatrix} p-\lambda & q \\ q & p-\lambda \end{vmatrix} = 0$		
	Characteristic equation for $\mathbf{M}$		
	$(p-\lambda)^2 - q^2 = 0$	**M1**	Forms correct characteristic equation
	$p-\lambda = \pm q$	**m1**	Takes square root of both sides and shows two solutions are obtained
	$\lambda = p \pm q$		
	$\lambda_1 = p+q$	**A1**	At least one eigenvalue correct
	$\lambda_2 = p-q$	**A1**	Both correct, either way around
	Subtotal	**4**	

Q	Answer	Marks	Guidance
4(b)	For $\lambda_1 = p+q$ $\begin{bmatrix} p & q \\ q & p \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = (p+q)\begin{bmatrix} x \\ y \end{bmatrix}$ $\begin{bmatrix} px+qy \\ qx+py \end{bmatrix} = \begin{bmatrix} (p+q)x \\ (p+q)y \end{bmatrix}$ $y = x$ $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ For $\lambda_2 = p-q$ $\begin{bmatrix} p & q \\ q & p \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = (p-q)\begin{bmatrix} x \\ y \end{bmatrix}$ $\begin{bmatrix} px+qy \\ qx+py \end{bmatrix} = \begin{bmatrix} (p-q)x \\ (p-q)y \end{bmatrix}$ $y = -x$ $\mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$	**M1** **A1** **A1**	Correct equation for determining an eigenvector **oe** At least one correct eigenvector. Must correspond to the correct eigenvalue. Allow any scalar multiple Both eigenvectors correct. Both eigenvectors must correspond to the correct eigenvalues. Allow any scalar multiple If **M1 A0 A0** awarded, allow **SC1** for $\mathbf{v}_2$ associated with λ_1 and $\mathbf{v}_1$ associated with λ_2
	Subtotal	**3**	

Q	Answer	Marks	Guidance
4(c)	$\mathbf{U} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$	B1ft	**ft** their eigenvectors
	$\mathbf{D} = \begin{bmatrix} p+q & 0 \\ 0 & p-q \end{bmatrix}$	B1ft	**ft** their eigenvalues
	Subtotal	**2**	

	Question 4 Total	**9**	

Q	Answer	Marks	Guidance
5(a)(i)	$\alpha\beta + \beta\gamma + \gamma\alpha = 0$	**B1**	Correct value for $\alpha\beta + \beta\gamma + \gamma\alpha$ **PI** by later working
	$\alpha\beta\gamma = -\frac{4}{27}$	**B1**	Correct value for $\alpha\beta\gamma$ **PI** by later working
	$2\alpha\beta + \beta^2 = 0$ and $\alpha\beta^2 = -\frac{4}{27}$	**B1ft**	Both correct **ft** their $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$
	$-\frac{8}{27\beta} + \beta^2 = 0$	**M1**	Attempt to eliminate either α or β from their equations
	$\beta^3 = \frac{8}{27}$	**A1**	At least one of α^3 or β^3 correct
	$\left[\alpha = -\frac{\beta}{2} \Rightarrow\right] \alpha^3 = -\frac{1}{27}$		
	$\alpha = -\frac{1}{3}$ and $\beta = \frac{2}{3}$	**A1**	Correct value for α and β
	Subtotal	**6**	

Q	Answer	Marks	Guidance
5(a)(ii)	$\alpha + \beta + \gamma \left[= -\frac{1}{3} + \frac{2}{3} + \frac{2}{3}\right] = 1$		
	$\alpha + \beta + \gamma = -\frac{k}{27}$		
	$k = -27$	**B1**	
	Subtotal	**1**	

Q	Answer	Marks	Guidance
5(b)	$27(1-\mathrm{i})^3 + k(1-\mathrm{i})^2 + 4 = 0$	**M1**	Substitutes $\alpha = 1-\mathrm{i}$ into cubic equation
	$27(-2-2\mathrm{i}) + k(-2\mathrm{i}) + 4 = 0$	**M1**	Simplifies $(1-\mathrm{i})^3$ and $(1-\mathrm{i})^2$
	$k = -27 + 25\mathrm{i}$	**A1**	**CAO**
	Subtotal	**3**	

	Question 5 Total	**10**	

Q	Answer	Marks	Guidance
6(a)	$\mathrm{f}(k+1) = 2^{4k+7} + 3^{3k+4}$	B1	Correct expression for $\mathrm{f}(k+1)$ with at least the powers simplified
	$2^{4k+7} = 2^4 \times 2^{4k+3} = 16 \times 2^{4k+3}$	E1	Shows this line in their working
	$\mathrm{f}(k+1) - 16\mathrm{f}(k)$		
	$= \left(16 \times 2^{4k+3} + 3^{3k+4}\right) - 16\left(2^{4k+3} + 3^{3k+1}\right)$		
	$= 3^{3k+4} - 16 \times 3^{3k+1}$		
	$= 81 \times 3^{3k} - 48 \times 3^{3k}$		
	$= (81-48) \times 3^{3k}$		
	$= 33 \times 3^{3k}$	A1	**AG** Must be convincingly shown **M1 E0 A1** is possible
	Subtotal	**3**	

Q	Answer	Marks	Guidance
6(b)	$\mathrm{f}(1)\ \left[=2^7+3^4=209\right]=19\times11$ [Therefore, true for $n=1$]	**B1**	
	Assume true for $n=k$ (★)		
	$\mathrm{f}(k+1)=16\mathrm{f}(k)+33\times3^{3k}$	**M1**	Relates $\mathrm{f}(k+1)$ to $\mathrm{f}(k)$ using their answer to **part (a)**
	$\mathrm{f}(k+1)=11M+11N$		
	$\mathrm{f}(k+1)=11(M+N)$		
	Therefore, $\mathrm{f}(k+1)$ is a multiple of 11	**A1**	
	Hence, true for $n=k+1$ (★★)		
	and since true for $n=1$ (★★★)		
	By induction $\mathrm{f}(n)$ is a multiple of 11 for all integers $n\geq1$ (★★★★)	**E1**	Must have (★), (★★), (★★★) present, previous 3 marks awarded and a final statement (★★★★) clearly indicating that it relates to all integers $n\geq1$ and induction
	Subtotal	**4**	

	Question 6 Total	**7**	

Q	Answer	Marks	Guidance
7(a)	$\dfrac{4r^2+16r+14}{(r+1)(r+2)(r+3)}$ $=\dfrac{A}{r+1}+\dfrac{B}{r+2}+\dfrac{C}{r+3}$ $4r^2+16r+14 = A(r+2)(r+3)$ $+B(r+1)(r+3)$ $+C(r+1)(r+2)$ When $r=-1$ $2=2A \Rightarrow A=1$ When $r=-2$ $-2=-B \Rightarrow B=2$ When $r=-3$ $2=2C \Rightarrow C=1$	**M1**	Forms at least two correct equations in A, B or C or two correct simultaneous equations in A, B and C
	$\dfrac{4r^2+16r+14}{(r+1)(r+2)(r+3)}$ $=\dfrac{1}{r+1}+\dfrac{2}{r+2}+\dfrac{1}{r+3}$	**A1**	Correct partial fractions
	Subtotal	**2**	

Q	Answer	Marks	Guidance
7(b)	$\sum_{r=1}^{2n} \frac{(-1)^r\left(4r^2+16r+14\right)}{(r+1)(r+2)(r+3)}$		
	$= -\frac{1}{2}-\frac{2}{3}-\cancel{\frac{1}{4}}$		
	$+\frac{1}{3}+\cancel{\frac{2}{4}}+\cancel{\frac{1}{5}}$		
	$-\cancel{\frac{1}{4}}-\cancel{\frac{2}{5}}-\frac{1}{6}$	**M1**	Writes at least three lines of subtracting terms, using their answer to **part (a)**
	$\vdots$		
	$+\frac{1}{2n-1}+\cancel{\frac{2}{2n}}+\cancel{\frac{1}{2n+1}}$		
	$-\cancel{\frac{1}{2n}}-\cancel{\frac{2}{2n+1}}-\frac{1}{2n+2}$		
	$+\cancel{\frac{1}{2n+1}}+\frac{2}{2n+2}+\frac{1}{2n+3}$	**M1**	Writes at least three consecutive lines, clearly showing the relevant cancellations
	$= -\frac{1}{2}-\frac{1}{3}+\frac{1}{2n+2}+\frac{1}{2n+3}$	**M1**	Correctly reduces the expression to four terms
	$= -\frac{5}{6}+\frac{1}{2(n+1)}+\frac{1}{2n+3}$		
	$= \frac{-10n^2-13n}{6(n+1)(2n+3)}$	**A1**	Obtains the correct final answer in the correct form. Must have been awarded the previous three **M1** marks. Condone $-\frac{10n^2+13n}{6(n+1)(2n+3)}$
	Subtotal	**4**	

Q	Answer	Marks	Guidance
7(c)	$\sum_{r=1}^{\infty} \frac{(-1)^r \left(4r^2+16r+14\right)}{(r+1)(r+2)(r+3)}$ $= \lim_{n\to\infty}\left(\frac{-10n^2-13n}{6(n+1)(2n+3)}\right)$ $= -\frac{5}{6}$	**B1ft**	**ft** their $\frac{a}{2c}$
	Subtotal	**1**	

	Question 7 Total	**7**	

Q	Answer	Marks	Guidance
8(a)	$(2x+ky-z)+(3x-2y+z)=3+5$	M1	Adds together the equations of the two planes
	$y=\dfrac{8-5x}{k-2}$	A1	Finds one variable in terms of another **oe**
	$z\left[=5-3x+2y\right]=\dfrac{5k+6-(3k+4)x}{k-2}$		
	Point on *L* where $x=0$		
	$\begin{bmatrix}0\\ \dfrac{8}{k-2}\\ \dfrac{5k+6}{k-2}\end{bmatrix}$	A1	Finds a correct point on *L* **PI**
	Direction vector for *L*		
	$\begin{bmatrix}1\\ -\dfrac{5}{k-2}\\ -\dfrac{3k+4}{k-2}\end{bmatrix}$	B1	Correct direction vector for *L* Allow any multiple of $\begin{bmatrix}1\\ -\dfrac{5}{k-2}\\ -\dfrac{3k+4}{k-2}\end{bmatrix}$ **PI**
	$\mathbf{r}=\begin{bmatrix}0\\ \dfrac{8}{k-2}\\ \dfrac{5k+6}{k-2}\end{bmatrix}+\lambda\begin{bmatrix}1\\ -\dfrac{5}{k-2}\\ -\dfrac{3k+4}{k-2}\end{bmatrix}$	A1ft	Correct equation in the correct form **ft** their point on the line and their direction vector
	Subtotal	**5**	

Q	Answer	Marks	Guidance
8(b)(i)	$12(\lambda)-\left(\frac{8}{k-2}-\frac{5\lambda}{k-2}\right)$ $-\left(\frac{5k+6}{k-2}-\frac{\lambda(3k+4)}{k-2}\right)=40$	M1	Substitutes the general point of their *L* into the equation for Π_3 **oe**, such as forming a correct determinant set equal to zero
	$\frac{1}{k-2}\left(12\lambda(k-2)-8+5\lambda\right.$ $\left.-5k-6+\lambda(3k+4)\right)=40$		
	$\frac{\lambda(15k-15)-14-5k}{k-2}=40$	A1	Correct and simplified linear equation in a parameter or a correct and simplified linear equation in k from determinant set equal to zero
	[No unique solution if] k = 1	A1	**CAO**
	Subtotal	**3**	

Q	Answer	Marks	Guidance
8(b)(ii)	[The three planes forms a triangular] prism	E1	Correct geometrical interpretation
	Subtotal	**1**	

	Question 8 Total	**9**	