OXFORD AQA
INTERNATIONAL QUALIFICATIONS

INTERNATIONAL A LEVEL
FURTHER MATHEMATICS

Further Pure Mathematics 4

Specimen paper 0X:00 GMT Time allowed: 1 hour 15 minutes

Materials
- For this paper you must have the OxfordAQA Pure Mathematics Examination Booklet (enclosed)
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.
- The final answer to questions requiring an algebraic expression should be given in simplified form, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 60.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise, marks may not be awarded.

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Answer **all** questions in the spaces provided.

1 (a) The matrix $\mathbf{M}_T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -0.6 & -0.8 \\ 0 & 0.8 & -0.6 \end{bmatrix}$ represents the transformation T

Describe fully the single transformation T represented by the matrix $\mathbf{M}_T$

[3 marks]

1 (b) The matrix $\mathbf{P} = \begin{bmatrix} 4 & -k & 2 \\ 2 & 1 & 3 \\ -3 & 0 & k \end{bmatrix}$ where k is a constant.

It is given that $\det \mathbf{P} > 0$

Find the range of values for k

[4 marks]

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Answer _____

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2 The vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are given by

$$\mathbf{a} = 4\mathbf{i} - 28\mathbf{j} + p\mathbf{k}, \quad \mathbf{b} = 5\mathbf{i} + 3\mathbf{j} - 2\mathbf{k} \quad \text{and} \quad \mathbf{c} = 2\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$$

where p is a constant.

2 (a) In the case where $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are coplanar vectors, use a scalar triple product to find the value of p

[3 marks]

$p =$ _____

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2 (b) In the case where **a**, **b** and **c** are mutually perpendicular, find the value of p

[3 marks]

$p =$ _____

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3 The integral I is defined as

$$I = \int \mathrm{e}^{2x} \sin(3x)\, \mathrm{d}x$$

3 (a) Show that

$$I = \mathrm{Im}\left(\int \mathrm{e}^{(2+3\mathrm{i})x}\, \mathrm{d}x \right)$$

[1 mark]

3 (b) Use the result given in **part (a)** to find I

Give your answer in a form that does **not** involve complex numbers.

[4 marks]

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Answer _____

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4 The matrix $\mathbf{M}$ is given by

$$\mathbf{M} = \begin{bmatrix} p & q \\ q & p \end{bmatrix}$$

where p and q are constants and $q \neq 0$

The eigenvectors of $\mathbf{M}$ are $\mathbf{v}_1$ and $\mathbf{v}_2$

The corresponding eigenvalues of $\mathbf{M}$ are λ_1 and λ_2

4 (a) Find λ_1 and λ_2 in terms of p and q

[4 marks]

$\lambda_1 =$ _____ $\lambda_2 =$ _____

4 (b) Find $\mathbf{v}_1$ and $\mathbf{v}_2$

[3 marks]

$\mathbf{v}_1 =$ _____ $\mathbf{v}_2 =$ _____

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4 (c) Write down a matrix $\mathbf{U}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{M} = \mathbf{UDU}^{-1}$

[2 marks]

$\mathbf{U} =$ _____ $\mathbf{D} =$ _____

Turn over for the next question

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5 The numbers α, β and γ are the roots of the cubic equation

$$27z^3 + kz^2 + 4 = 0$$

where k is a constant.

5 (a) (i) In the case when $\beta = \gamma$, find the value of α and the value of β

[6 marks]

$\alpha =$ _____ $\beta =$ _____

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5 (a) (ii) In the case when $\beta = \gamma$, find the value of k

[1 mark]

$k =$ _____

5 (b) In the case when $\alpha = 1 - \mathrm{i}$, find k

[3 marks]

$k =$ _____

Turn over for the next question

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6 The function f is defined as $\mathrm{f}(n) = 2^{4n+3} + 3^{3n+1}$ for $n \in \mathbb{N}$

6 (a) Show that $\mathrm{f}(k+1) - 16\mathrm{f}(k)$ can be expressed in the form $b \times 3^{3k}$ where b is an integer.

[3 marks]

6 (b) Prove by induction that $\mathrm{f}(n)$ is a multiple of 11 for all $n \in \mathbb{N}$

[4 marks]

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7 (a) Express $\dfrac{4r^2+16r+14}{(r+1)(r+2)(r+3)}$ in partial fractions.

[2 marks]

Answer _____

7 (b) Use your answer to **part (a)** with the method of differences to find

$$\sum_{r=1}^{2n} \frac{(-1)^r\left(4r^2+16r+14\right)}{(r+1)(r+2)(r+3)}$$

Give your answer in the form

$$\frac{an^2+bn}{c(n+1)(2n+3)}$$

where a, b and c are integers.

[4 marks]

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Answer _____

7 (c) Use your answer to **part (b)** to find the value of

$$\sum_{r=1}^{\infty} \frac{(-1)^r \left(4r^2 + 16r + 14\right)}{(r+1)(r+2)(r+3)}$$

[1 mark]

Answer _____

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8 The planes Π_1, Π_2 and Π_3 have the Cartesian equations

$$\begin{aligned} \Pi_1 &: 2x + ky - z = 3 \\ \Pi_2 &: 3x - 2y + z = 5 \\ \Pi_3 &: 12x - y - z = 40 \end{aligned}$$

where k is a constant.

The line of intersection of Π_1 and Π_2 is L

8 (a) Find a vector equation of L

Give your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{d}$ where $\mathbf{a}$ and $\mathbf{d}$ are column vectors in terms of k

[5 marks]

Answer _____

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8 (b) It is given that L does **not** intersect Π_3

8 (b) (i) Find the value of k

[3 marks]

$k =$ _____

8 (b) (ii) Give a geometric interpretation of the arrangement of Π_1, Π_2 and Π_3

[1 mark]

END OF QUESTIONS

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